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Mortality linked securities and derivatives
- a way to tackle the longevity risk?

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"By providing financial protection against the major 18th and 19th century risk of dying too soon, life insurance became the biggest financial industry of that century, growing profitably world-wide for more than 150 years, i.e. until 1914. Providing financial protection against the new risk of not dying soon enough may well become the next century's major and most profitable industry."

Peter Drucker in:

Financial Services, The Economist, Sept. 25th 1999

1 Introduction

Pension systems in developed countries suffer more and more from the interaction of a decreasing birth rate on the one hand and a increasing life expectancy on the other hand. Hence governments are confronted with massive financial problems on both sides, i.e. revenue and expenditure. The increasing longevity of retirees and the decreasing birthrate, lead, a constant labour force participation rate and a unchanged pension entrance age supposed, to a increasing number of retirees and to a decreasing number of contributors, see Leinert und Wagner (2001). The arising problems affect not only governments, the problems affect also employers retirement plans as well as private insurance companies and last but not least every individual.

The impact of one determinant, i.e. the increasing life expectancy, on private insurance business and the different ways to transfer this risk to the capital markets is the underlying subject of this master thesis. The regional focus of this thesis is the Austrian market, further it is structured in an historical overview and an empirical part to verify the usefulness of such instruments.

The aim of this thesis was at first to introduce the reader in this new market through a historical refurbishment. The second task was to model the mortality rates of 65 year old Austrian males and sample them afterwards 5000 times through a Monte Carlo simulation. The 5000 simulated mortality rates of 65 year old Austrian males should then be the dataset for all further calculations.

Third, the usefulness of two different hedging tools should be evaluated. To analyze those instruments the calculus of Net Present Values was chosen, whereas a conditional probability was introduced to calculate the so called Expected Net Present Value. As both instruments delivers a future payment or payments this decision seems reasonable to verify the usefulness of those instruments..

The thesis is structured in the following way. The first chapter gives an general overview over the topic of longevity risk. The impact of longevity on pension schemes and private insurance business is briefly discussed. Further

the participants and their role in the market of mortality linked securities and derivatives are introduced.

The second chapter gives an overview from historical to recent developments to tackle longevity risk. Especially the hedging possibilities via capital markets as well as traditional methods are explained here. The main focus is on two instruments. The first with a bond like structure, in fact the European Investment Bank (EIB) bond, and second, one similar to a zero coupon swap, called q-forward and provided by JP-Morgan. The third chapter provides an brief overview of mortality linked indices.

With the fourth chapter starts the empirical part, where at first some stylized facts on longevity in Austria are introduced and afterwards forecasting and sampling of mortality rates is treated. As excess to this topic, the Lee-Carter Model (Lee and Carter 1992) is introduced. Constitutive on Lee-Carter, the "Functional Data Approach" to model mortality rates is discussed in this chapter. In fact the functional data model provided by Hyndman and Ullah (2006) was used, as the mean squared forecasting error of this model is superior to Lee and Carter, see Hyndman and Ullah (2006).

In chapter five and six the two hedging instruments was applied to the simulated dataset. In the former the longevity bond of the European Investmentbank (EIB), designed for 65 year old British and Welsh males, was analyzed. The reason therefore was; it was the first publicly announced longevity instrument, however it was drawn back from the market through a lack of interest. Thus chapter five should give an answer whether the bond would have been a good choice for Austria or not.

The latter was JP-Morgans q-forward as counterpart. The reason therefore was that trades with a q-forward happend, unlike to the EIB-bond. In fact the first hedge with a q-forward took place in February 2008, see Biffis and Blake 2009. The analysis should also give an answer to the question, whether the q-forward is a reasonable hedging tool for 65 year old Austrian males or not. In chapter seven the findings of the analysis are concluded. Appendix A gives a brief introduction in the calculation of mortality tables and appendix B shows a fraction of the R-Code.

1.1 Pension Schemes

Overall the pension schemes could be classified into two major types, *funded* or *unfunded* schemes, depending on the determination of the retirement payments. A funded scheme requires for every employed an individual capital account to accumulate contributions during the time employed and to distribute the contributions during retirement. The contributions are invested in different assets, thus the retirement payment depends on the performance of the assets chosen. Obviously the height of the retirement payment is not known in advance, hence there exists no guarantee that obtained benefits will match the necessary amount of money to keep welfare during retirement at an appropriate level.

The unfunded scheme, used in Austria, is also known as "pay as you go" scheme (PAYGO), "generations contract" or as "Bismarck Type". Bismarck invited this system in 1889 at which the pension payments for todays retirees are immediately financed by todays labourers. In fact this means, that the younger generation is ruled by law to provide pension payments for the elder generation. The unfunded schemes contributions depends only on the height of the employees salary. The height of the benefits, if retired, depends thus on the contribution, the duration of employment and the underlying legal requirements to determine the benefits, see Felbinger et al. (2007).

1.2 Life Insurance products

There are three main types of contracts to distinguish: the *term insurance*, the *pure endowment* and the *annuity* contract. Obviously more complex contract designs are possible by combining these three types among each other as discussed below. Further it is possible to confine benefit payments to predetermined conditions, for instance a lifelong annuity if occupational invalidity occurs or death benefits only for predetermined causes of death.

- The *term insurance* contract of duration n is an agreement to pay the sum insured if the insured, a life aged x , dies at any time during $x + n$, i.e. the term of the policy, with payment to be made within the year

death happens. The sum insured is predetermined at the contracts inception, only the time of death, i.e. the date of payment is random, see Gerber (1986). The increasing life expectancy is a welcome effect for the term insurance business because the risk for the insurance company arises if the policy holder dies within the term of the policy, hence a insured surviving the whole term period minimizes this inherent risk.

- The *pure endowment* is the second type discussed. This type of contract pays a predetermined sum insured at the end of n policy years, if the policyholder initially aged x survives to age $x + n$. This contract is rather simple since neither the time of payment nor the amount is uncertain.

The major part of contracts in action are *endowments*. An endowment is a combination of a term insurance and a pure endowment, thus paying the sum insured if the life aged x dies within the term of the policy or otherwise at the end of the n -th year, see Gerber (1986). Obviously the endowment is affected by longevity in the same way as the term insurance, hence an increasing life expectancy increases the probability that contributions are paid over the whole term of the policy.

- The *annuity contract* is an agreement to pay a scheduled payment to the policyholder at a predetermined date within a year, usually as long the annuitant is alive. The payments are fixed at the contracts inception and stop with the random time of death. Concerning annuity contracts, there are two types to distinguish, namely the deferred annuity and the immediate annuity. In case of the former the policy holder has the possibility to contribute to the contract within the deferred term or in the case of the latter the annuitant pays a lump sum before the inception to receive scheduled payments afterwards, see Gerber (1986).

Further there exists the possibility to use these contracts for employment based pensions (- or for financial precaution of the bereaved), called "Betriebliche Altersvorsorge" (BAV) (- or "Betriebliche Hinterbliebenen Vor-

sorge") in Austria. As incentive for employers and employees to set up such additional plans, the government created tax advantages. For the employee the advantage is that the contribution to the capital account happens before the income tax appears, thus double taxation (income tax and capital gains tax) is avoided. For the employer the advantage is the reduction of the non wage labour costs. The gross salary is the calculation basis for the non wage labour costs and is thus reduced by the amount, contributed to the individual capital account. Further counts the contribution as running cost and is thus minimizing the employers income tax, see Felbinger (2006).

1.3 About the risk

The PAYGOs inherent principle requires a sufficient level of contributors to the PAYGO scheme. In Austria there was , 435 retired persons per 1000 contributors, in 1966 and 621 retired persons per 1000 contributors in 2006¹. This number suggests that PAYGO schemes provided by governments enter into financial difficulties. However, the level of contributors is only related to PAYGO schemes. The second determinant, i.e. the increasing longevity, is far more widespread. The increasing longevity combined with a declining labour force creates a steadily increasing shortage between contributions and benefits. In 1990 there was a shortage of 3.82 billion Euros followed by a shortage of 6.78 billion Euros in 2004². Obviously this shortages must be financed with other tax money. To overcome this development Austrias government started pension reforms in 2003, 2004 and 2005. The aim was to reduce the new number of retirees, i.e. expand the duration employed and further decrease in the long term the benefits. The shortage in 2005 was 5.23 billion Euros and in 2006 5.36 billion Euros³. The numbers suggest that the reforms work, at least in the short term. But whether this shortages are financeable in the long run or not, might at this time nobody to answer.

This development creates a shift to private pension prevention to keep the monetary welfare during retirement at an appropriate level and/ or to permit

¹Source: Hauptverband der Sozialversicherungsträger

²Source: Hauptverband der Sozialversicherungsträger

³Source: Hauptverband der Sozialversicherungsträger

an earlier retirement age. Thus the private insurance industry is more and more confronted by individuals trying to hedge their own longevity exposure to the insurer, i.e. the lifelong annuity provider.

Hence the risk for the insurance industry lies in the mortality rates q_x , which are part of the formulas determining the lifelong annuity payment fixed at inception. In fact the inherent risk of annuity contracts is that the increasing longevity, i.e. decreasing q_x , creates a uncertainty whether realized future mortality rates fall below the mortality rates anticipated by annuity providers in the past, or not. If the future mortality rates are far more lower than anticipated in the past, than the actual expenditures will overshoot the aggregated premiums.

In recent literature this kind of risk is called *longevity risk*. This uncertainty in future developments constitutes an enormous risk factor for involved institutions because the outstanding annuities are not less than liabilities for the enterprises. If an enterprise applies the IFRS accounting rules the changes in market value of the liabilities have to go through the income statement. Obviously this liability item makes the balance sheet vulnerable. The corollary in the sense of the shareholder value maximization principle is that affected enterprises seek to get rid off this items.

The funded schemes, as mentioned above, are only affected by longevity risk and not by the declining labour force. This is obvious as every employee pays on his own capital account. In fact the problem is the calculation of the right height of benefit payments, because nobody knows his residual lifetime. Hence the accumulated contributions will be enough for the entire retirement period if one overestimates his residual life time or there will be a shortage if one underestimates his residual lifetime. In the latter case the individual is a victim of longevity. Further the impact of high inflation rates is far more greater. The PAYGO pensions are usually indexed to inflation, but the individual payment from the capital account not. One possibility to overcome this problems is to buy a lifelong annuity with the accumulated contributions from the private insurance industry.

1.4 First impacts of longevity risk

In December 2000 this topic aroused public interest after the world's oldest life insurer the Equitable Life Assurance Society (ELAS), was forced to close for new business. During the period of 1957 and 1988 ELAS had sold with profit pension annuities with "guaranteed annuity rates" fixed by reference to specific assumptions regarding interest rates and life expectancy. These guarantees became very valuable in the 1990s due to a combination of falling interest rates and a decreasing mortality rate, and it was the rise in the values of this guarantees that led to ELAS financial difficulties. These could have been avoided if ELAS had hedged its exposure to both interest rate risk and longevity risk, but for years ELAS failed to appreciate the extent of its financial exposure. The failure of ELAS to do so bespeaks of the poor state of interest rate and longevity risk management in the Society. However, even if it had anticipated the problem, it still lacked good instruments to hedge its exposure to both risks, particularly longevity (see Blake et al., 2006).

Bowe (et al., 2006) figured out that in Germany the improvement of the actuarial reserve fund due the adoption of new mortality tables (DAV1994R) in the year of 1995 showed the German Life insurers, the more or less unexpected risk of longevity. The actuarial reserve fund ${}_mV_x$ is defined as the difference between the present value of future benefit payments $E(Z_m^b)$ and the present value of future premium payments $E(Z_m^c)$ calculated at the beginning of year m , given that the customer has survived this date:

$${}_mV_x = E(Z_m^b) - E(Z_m^c) \quad (1)$$

Because of the young stock and the calculation with the former actuarial interest rate of 4% the reserve improvement was quite moderate. Also the possible extension over a period of eight years helped the insurers to overcome the adoptions without big difficulties.

At the adoption of a new actuarial basis on the new mortality table DAV2004R a reserving requirement of 8 billion Euro for the whole insurance

sector became apparent. In the face of the down melted surpluses and small interest profits this volumina, would had been only financeable, for a few insurers within one business year. Finally this yielded to an extension of the reserving procedure over 20 years.

Additionally the new regulatory regime for insurance companies operating in the European Union (EU), Solvency II, which will be introduced in 2012, can make the situation even more acute if longevity risk cannot be hedged effectively or marked to market⁴ as Blake, Boardman and Cairns (2010) proposed. However, if the current Solvency II proposals will be adopted, insurers will be required to hold significant additional capital to back their annuity liabilities thus an additional increase in ${}_mV_x$, i.e. the reserves, will be required.

⁴A measure of the fair value of accounts that can change over time, such as assets and liabilities. Mark to market aims to provide a realistic appraisal of an institution's or company's current financial situation, i.e. the financial statement date

1.5 Who participates in markets for mortality – linked securities ?

Obviously it is necessary or helpful to know something about the markets participants before examining the products developed. For that reason I will briefly introduce the stakeholders of these markets, see Blake et al. (2006).

1.5.1 The Hedgers

Like in other markets (i.e. currency or interest market), the Hedgers are the main component. In the market for mortality linked securities the Hedgers are exposed to longevity risk and search for possibilities to lay off that risk. One way to handle this risk for parties with unwanted exposure to longevity risk is to pay a premium to a counterparty in order to lay off a part of this risk. For life insurance companies two ways come into consideration: reinsurance, or transferring the risk to the capital markets.

Reinsurer usually take over excess exposure to several risks from insurers. The reason therefore is that they get a bigger diversification if they deal with many primary insurers when they lay off their risks to the reinsurer. Keeping the "Law of large numbers" in mind, it is obvious that the reinsurer is able to calculate the premiums more precisely. Reinsurers thus cover the excess liability or cover the full risk of contracts. Given the higher number of contracts, reinsurer understand the covered risks better than the primary insurer. They have the capability to study several exposures to be aware of losses. Probably that's the reason why reinsurers are longevity averse, i.e. cover long term longevity risk. If one thinks about the development of gene technology this behavior is not devious⁵.

1.5.2 Speculators and Investors

The market for mortality linked securities might attract speculators and investors. Speculators trade under the purpose of generating profits out of the price fluctuations of securities. Investors instead miss the trading purpose,

⁵http://media.swissre.com/documents/rethinking_ageing_longevity_life_insurance_in_light_of_discovery_of_genes_controlling_longevity_en.pdf

they decide to buy a security under the preassigned claim on future interest - or dividend payments. Thus the buy is not related to trade for investors but entirely on the intrinsic value of the share, i.e. it is geared to the regular income from investment, see Schumpeter (1954).

The active participation of speculators is necessary for liquidity, essentially in futures and options market. Overall they guarantee the permanent possibility to buy and sell in any market. The expected returns from mortality linked securities have a low correlation with standard financial products, hence they might provide an attractive opportunity to diversify a portfolio of potential investors like hedge funds or investment banks. Despite this facts the market is actually too small for speculators. In particular there exists no standardized market for mortality linked securities until know.

1.5.3 Regulators

In Austria the “Finanzmarktaufsicht - FMA⁶” is the ruling authority concerning financial markets. The aims are :

- i) Guarantee the stability of the Austrian Financial Market
- ii) To strengthen confidence in a well functioning Austrian Financial Market
- iii) To protect creditors, investors and consumers under the actual rule of law
- iv) To act preventive concerning the compliance of supervisory norms, and to avoid offences

The major step to keep up the liquidity of annuity payers the FMA⁷ asked the insurer to foster reserves for all liquid annuities. They have to be sufficiently funded on the basis of the annuity table AVOE 2005M/F. Additionally a lump sum reserve for existing accrued future annuity rights of

⁶Mission statement of the „Finanzmarktaufsicht” – www.fma.gv.at

⁷Rundschreiben der FMA zur Nachreservierung von Rentenverträgen (GZ 9 000 110/7-FMA-II/1/05)

the closed annuity-generation was build and fitted on the 31 of December 2008 to the corresponding stock development. This action of the government seems to make sense. It protect the consumers, but for the annuity payers the strengthening of reserves could be very jeopardizing. Thus the government creates a hedge to the advantage of the consumer side and imposes the whole risk to the annuity payers. Figure 4 shows the development of the annuity tables the reader could draw his own conclusions on the development of the necessary reserving. Obviously annuity payers search for ways to get rid off this risk. Out of this we can conclude that longevity risk can affect annuity payers in two different ways, some are affected through the additional reserving, i.e. insurances, and some are affected through the increasing duration of payments, i.e. the government.

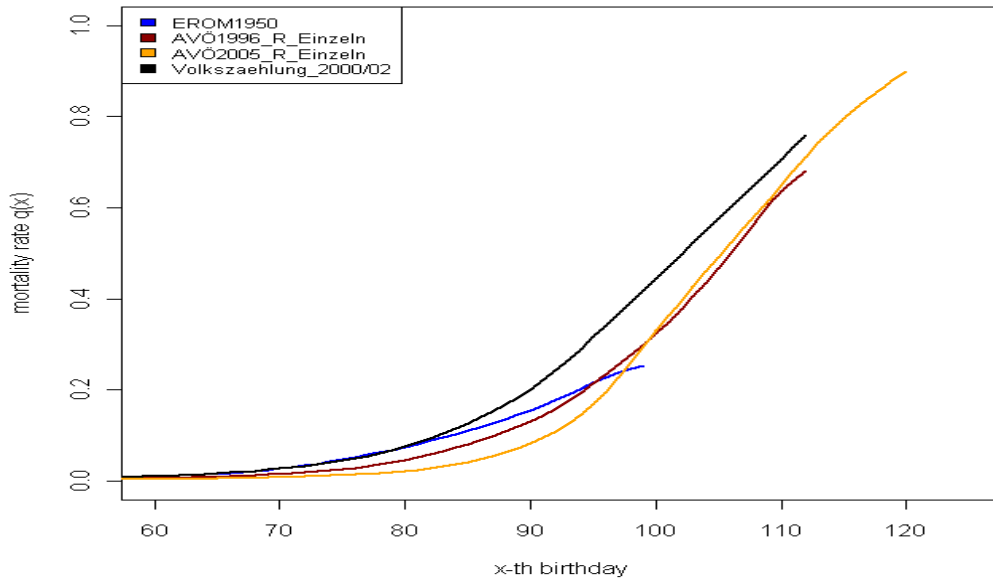


Figure 1: Different annuity tables compared to the 2000/02 mortality table
Data: AVOE and Statistik Austria

2 The "Life Market"

2.1 Standard solutions to bear the risk

This chapter gives an overview of currently known possibilities to bear the risk. It is based on Blake et al., (2006) and Biffis and Blake, (2009). Further it is expanded to meet the Austrian legislation body for insurances and financial intermediaries.

- i) A naive method would be to accept the risk as a part of the business engaged in, and hopefully understand it well enough to be prepared for future events.
- ii) Concerning different products a insurance group can seek to exploit the opposite effects of running life insurance contracts and pure annuity business, this is also called natural hedging, see Cox and Lin (2005). Through their defined benefit pension liabilities, pension funds are so called short longevity, as their liabilities rise with longevity. Instead the life insurance is long longevity as their liabilities fall with mortality. Natural hedging uses this windfall profits to balance aggregated liability cashflows. For Austria there must be mentioned that the FMA request the insurers to pay a certain amount of the profit out of the capital insurance business to the policy holder⁸. In fact there must be a payment of 85 percent of the obtained profits to the policy holder. Thus the amount available for natural hedging is 15 percent of the obtained capital insurance profits. Further capital insurance and annuity business are handled as different balance sheet items in life insurance business. Aggregated overall Austrian insurance companies and averaged over 2007 up to 2009, the insurance sum of the annuity business is about 47 percent of the capital insurance business⁹. A increasing longevity anticipated, the direction of the subventions is clear; the capital insurance would support the annuity business with much bigger

⁸Gewinnbeteiligungs-Verordnung-FMA(2006)-398

⁹Source: Jahresbericht 2009, Versicherungsverband Österreich

subventions as vice versa. This was reason enough for the FMA to prohibit cross subventions between this different balance sheet items. Hence, as long as this unequilibrium exists, natural hedging is at least for Austria a more or less theoretical concept.

- iii) Huge companies often run their own pension plan as additional benefit for their employees. If the plan members residual lifetime increases more than once expected, the plan or even the whole company will enter into financial problems. Obviously companies seek to get rid off this commitments, Biffis and Blake (2206) describes the pension buyout in the following way:" A typical example is represented by a company with assets A and liabilities L , valued by the plan actuary. When the plan's assets are insufficient to cover the liabilities, i.e. $A < L$, the company recognizes a deficit of $L - A$. When $A > L$ instead, the company's plan has a surplus of $A - L$. Life insurers are usually required to value liabilities under more prudent assumptions (on future mortality improvements, inflation rates, and market yields) than pension plans, resulting in a valuation $\tilde{L} > L$ for the liabilities. This increases reported deficits or reduces reported surpluses when a company approaches an insurer for transferring its pension assets and liabilities. In the case of a deficit, a company borrows the amount $\tilde{L} - A$ and pays it to an insurer to buyout its pension assets and liabilities. The transaction allows the employer to off-load the pension liabilities from its balance sheet. This means that the volatility of assets and liabilities associated with the pension plan accounts, the payment of management fees on the plan's assets, and any levies charged for members' protection insurance¹⁰ can be avoided. If buyout costs are financed by borrowing, a regular loan replaces pension assets and liabilities on the balance sheet. From the point of view of the plan members, the pensions are secured in full, subject, of course, to the solvency of the life insurer." For instance the "Financial Times" reports in February 2010, that the BMW Group wants to transfer 2.8 billion EUR out of their british pension fund to

¹⁰E.g., the Pension Protection Fund (PPF) in the UK.

"Abbey Life", the corresponding division of "Deutsche Bank", and to the London based insurer "Paternoster".

2.2 First steps towards capital markets

In this chapter I will introduce the first steps towards capital markets to hedge longevity risk. The first instruments had all a bond¹¹ like structure. Blake and Burrows (2001) were the first to introduce this proposal. They argue that bonds whose future coupon payments depends on the development of mortality could ease the insurance exposure to longevity risk. For a bond issued in x and maturity in $x + n$, for example, the coupons in $x + n$ depends on the fraction of people survived the $n - th$ year.

Swiss Re, a global reinsurance company, dared the first move in 2003, and launched a mortality catastrophe bond which will be described in the following subchapter as well as the long term bond of the European Investment Bank (EIB) and BNP Paribas¹² in 2004. Even though the Swiss Re bond is not a classical longevity bond, it is described here because of his role as an pioneer in the life market. A quite easy modification of this bond to fit the requirements of annuity payers is also included. This two instruments, with their strengths and weaknesses provide an instructive basis for further developments.

2.2.1 "Vita I" - the Swiss Re Mortality catastrophe bond

In December 2003, Swiss Re issued "Vita I", the first floating rate bond with a principal payment linked to a mortality index. The maturity of the bond was the 1st January 2007, i.e. a duration of three years. Blake, Cairns and Dowd (2008) stated that such short-dated mortality bonds are market-traded securities whose payments are linked to a mortality index. They are similar to catastrophe bonds. As such, they are designed to hedge brevity risk, rather than hedge longevity risk (the principal concern of this paper), but as an important successful example of a life market instrument, they are

¹¹Blake(2006) explained in "Pension Finance": Bonds are capital-market securities and as such have maturities in excess of one year. They are negotiable debt instruments. There are many different types of bonds that can be issued. The most common type is the *straight bond*. This is a bond paying a regular (usually semi-annual), fixed coupon over a fixed period to maturity or redemption, with the return of principal (that is, the par or nominal value of the bond) on the maturity date. All other bonds are variations on this.

¹²A French Investment Bank

included in this article.

Vita I was designed to hedge the reinsurers exposure to catastrophic mortality risks such as the Spanish flu in 1918 or terrorist attacks far greater than the attack on the World Trade Center in 2001. The issue size was \$400 million. The coupon payment for investors was set at three month U.S.Dollar LIBOR + 135 basis points (see Blake et. al. 2006). Cairns et. al.(2005) argued that for primary insurers and pension plans, the bond was only a hedge against one particular form of extreme short-term mortality risk. Pension funds which would be the beneficiaries of such an event because their pension liabilities would show a sudden drop after such significantly higher death rates, were , nevertheless prepared to buy the bond because it reduced variability in the asset-liability ratio and because the bond offered an attractive return relative to conventional bonds. Blake et al. (2006) describes the bond as follows:

"The principal is unprotected and depends on what happens to a specifically constructed index of mortality rates across five countries: the United States of America, U.K., France, Italy and Switzerland. The principal is repayable in full if the mortality index does not exceed 1.3 times the 2002 base level during any of the three years of the bond's life.

Payment at maturity (T)	$100\% - \sum_t loss_t$	if $\sum_t loss_t < 100\%$
	0%	if $\sum_t loss_t \geq 100\%$
Loss percentage in year t $= loss_t$	0%	if $q_t < 1.3q_0$
	$[(q_t - 1.3q_0)/(0.2q_0)] \times 100\%$	if $1.3q_0 \leq q_t \leq 1.5q_0$
	100%	if $1.5q_0 < q_t$
where:	q_0 = base index $q_t = \sum_j C_j \sum_i (G^m A_i q_{i,j,t}^m + G^f A_i q_{i,j,t}^f)$	
Key :	$q_{i,j,t}^m$ = mortality rate (deaths per 100,000) for males in the age group i for country j $q_{i,j,t}^f$ = mortality rate (deaths per 100,000) for females in the age group i for country j C_j = weight attached to country j A_i = weight attributed to age group i (same for males and females) G^m and G^f = gender weights applied to males and females respectively The following country weights apply: U.S.A. 70%, U.K. 15%, France 7.5%, Italy 5%, Switzerland 2.5%, male 65%, female 35%.	

Figure 2: Swiss Re mortality bond payoff schedule Source: *Blake et al.*,2006

The principal is reduced by 5% for every 0.01 increase in the mortality index above this threshold and is completely exhausted if the index exceeds 1.5 times the base level. The payoff schedule of the bond is shown in Figure 3 and Figure 4.

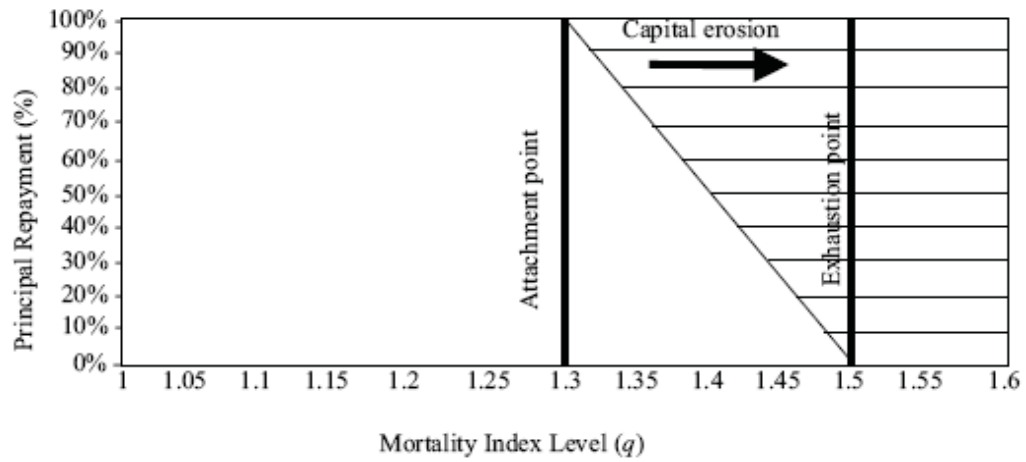


Figure 3: Terminal payoff of Swiss Re mortality bond to investors. Source: Blake *et al.*, 2006

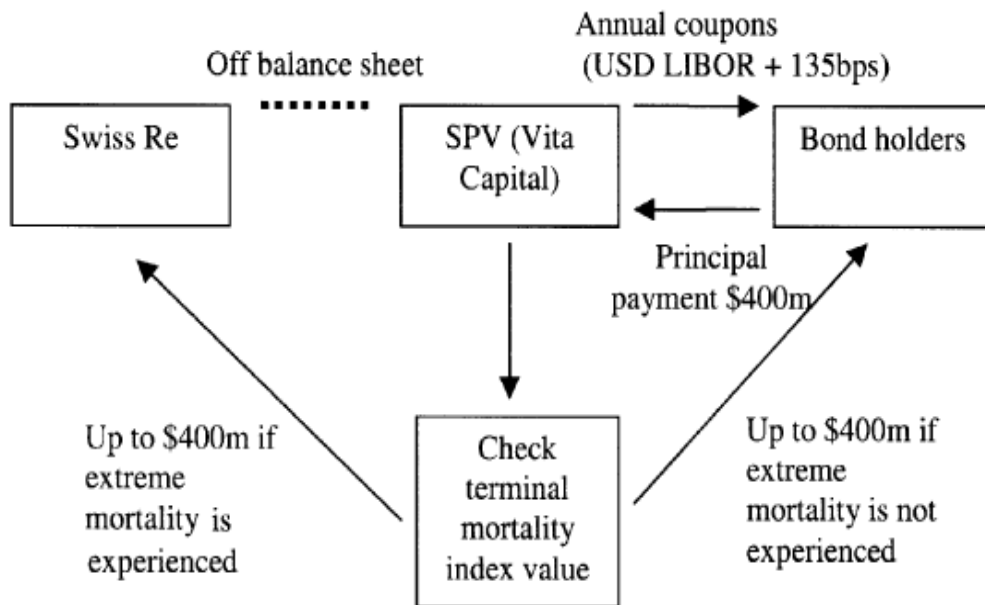


Figure 4: The structure of Swiss Re mortality bond. Source: Blake *et al.*, 2006

The bond was issued via a special purpose vehicle (SPV) called Vita Capital (VC). VC invested the \$400m principal in high-quality bonds and swaps the income stream on these for a LIBOR-linked cash flow. VC distributes the quarterly income to investors and any principal repayment at maturity. This structure is shown in Figure 2. The benefits of using an SPV in this context are that the cash flows are kept off balance sheet (which is good from Swiss Re's point of view) and that credit risk is reduced (which is good from the investors' point of view).

According to its 2004 annual report, life reinsurance is Swiss Re's primary source of business revenue, accounting for 30% of revenues, implying that profitability is negatively correlated with mortality rates. However, as the world's largest provider of life and health reinsurance, Swiss Re faces the potential difficulty of finding a sufficient number of counterparties on whom it can off-load this risk, and this has implications for its regulatory capital requirements. The bond therefore helps Swiss Re to unload some of the extreme mortality risk that it faces. It is also likely that Swiss Re was mindful of its credit rating and wanted to reassure rating agencies about its mortality risk management. Further, by issuing the bond themselves, Swiss Re are not dependent on the creditworthiness of other counterparties should an extreme mortality event occur. Thus, the bond gives Swiss Re some protection against extreme mortality risk without requiring that the company acquire any credit risk exposure in the process.

Investors in the bond take the opposite position and receive an enhanced return if an extreme mortality event does not occur. Some indication of how well compensated they were for taking on this extreme mortality risk arises from the work of Beelders & Colarossi (2004). They valued the bond using extreme value theory, assuming a generalized Pareto distribution for mortality. Recognizing that the terms of the bond are equivalent to a call option spread on the mortality index, with a lower strike price of $1.3q_0$ and an upper strike price of $1.5q_0$, Beelders and Colarossi estimated the value of the probability of attachment ($\text{prob}[q_t > 1.3q_0]$) at 33 basis points and the value of the probability of exhaustion ($\text{prob}[q_t > 1.5q_0]$) at 15 basis points. The expected loss on the bond was estimated to be 22 basis points, less than the

135 basis points of compensation on offer initially to Investors. Beelders and Colarossi concluded that the bond appeared to be a good deal for investors and in June 2004 the bond was trading at LIBOR+100 basis points. However, we should keep in mind that their figures are only estimates based on a model that ignores parameter uncertainty: plausible alternative parameter estimates can produce much higher values for the basis point compensation received by investors. Thus, we cannot be sure how good a deal the investors actually got. By November 2005 the mid-market price of the bond was equivalent to LIBOR+123 basis points. It is plausible (although we have no evidence for this) that this increase reflected the increased probability of a bird-flu pandemic in 2006.

The Swiss Re bond issue was fully subscribed and press reports suggest that investors were happy with it (e.g. Euroweek, 19 December 2003). These investors included a number of pension funds. These would have been attracted, in part, by the higher coupons being offered. They would also have been attracted by the hedging opportunities offered by the fact that the mortality risk associated with the bond is correlated with the mortality risk associated with active members of a pension plan. Specifically, consider an event that would trigger a reduction in the repayment of the Swiss Re bond. The large number of extra deaths would presumably extend to active members of the pension plan. Since death benefits are typically less than the pension liability for an individual member, the reduction in the value of the pension plan's Swiss Re bond investment would be matched by a reduction in the value of their plan liabilities. In the meantime, the bond offers a considerably higher return than similarly rated floating-rate securities. The bond's reception in the marketplace also suggests that investors believed the 135 basis points to represent a good deal.

In April 2005, Swiss Re announced that it had issued a second lifecatastrophe bond with a principal of \$362m, using a new SPV called VitaCapital II. The maturity date is 2010 and the bond was issued in three trenches: Class B (\$62m), Class C (\$200m) and Class D (\$100m). The principal is at risk if, for any two consecutive years before maturity, the combined mortality index exceeds specified percentages of the expected mortality level (120% for Class

B, 115% for Class C, and 110% for Class D). The bond was fully subscribed."

2.2.2 Modification of the Swiss Re Bond to hedge longevity risk.

As discussed in the introduction, the strengthening of reserves, required by the authorities, if longevity increases too much, jeopardizes the liquidity of insurers eminently. Hence insurers seek for ways to hedge this increase in policy reserves. A possibility could be a bond, build on the basis of the Swiss Re "Vita I" bond. The Swiss Re bonds structure reduces the principal by 5% for every 0.01 increase in the mortality index above this threshold and is completely exhausted if the index exceeds 1.5 times the base level.

Inspired by this mechanism a insurer could offer an bond where the repayment of the principal is linked to the difference between the mortality rates of the actual annuity table q_x^{actual} and the mortality rates calculated out of the next population census q_x^{future} . Like the Swiss Re bond "VitaI" two mortality thresholds q_x^a and q_x^b , $q_x^a > q_x^b$ are fixed below q_x^{actual} at the contracts conclusion, i.e. $q_x^{actual} > q_x^a > q_x^b$. In Austria the new tables are usually computed after a population census, hence the insurer knows more or less the date when the strengthening of reserves will happen or not. Obviously the insurer should set the maturity date of the bond align with this policy action. To make this bond interesting for investors the principal is invested in several AAA-ranked bonds and the income stream of this bonds is swapped for a LIBOR-linked cashflow like the "Vita I" Bond. At maturity the development of the mortality rates decides what happens to the principal:

- i) If $q_x^{future} > q_x^a$ the principal is fully paid back,
- ii) If q_x^{future} is in between $[q_x^a, q_x^b]$ the paid back principal reduces for a pre-defined percentage rate,
- iii) If $q_x^{future} < q_x^b$ the whole principal goes to the insurer.

Ideally the amount required by the authorities to foster reserves equals the deducted principal, so the insurer could get rid off longevity risk. However the investment risk still remains. One way to tackle investment risk would

be to divide it into a number of single risks, in this case the insurer should invest in several AAA-bonds instead of one.

2.2.3 The EIB/ BNP-Pariba longevity bond

In November 2004 , one year on from the issue of the Swiss Re bond, BNP Paribas announced that it had arranged for the EIB to issue a longevity bond that goes a very long way towards providing a solution for financial institutions looking for instruments to hedge their long-term systematic mortality risks. The total value of the issue is £540 million, and is primarily aimed at UK pension funds. The concept and usefulness of longevity bonds have been discussed for a number of years see Cox et al. (2000) and Blake & Burrows (2001). But it has taken time for the capital markets to develop the finer implementation details of these contracts (even though here the detail is relatively simple), and for both potential issuers and investors to decide that the time is right, see Cairns et al. (2005).

The bond itself was withdrawn after one year of marketing because it generated not enough demand. But it was the pioneering first step to deal with long-term longevity risk and it offers the opportunity to learn out of its shortcomings for future developments. The following description is based on Blake et al. (2006).

"2.2.3.1 This security was to be issued by the European Investment Bank (EIB), with BNP Paribas as the designer and originator and Partner Re as the longevity risk reinsurer. The face value of the issue was £540 million and the bond had a 25-year maturity. The bond was an annuity (or amortizing) bond with floating coupon payments, and its innovative feature was to link the coupon payments to a cohort survivor index based on the realized mortality rates of English and Welsh males aged 65 in 2002. The initial coupon was set at £50 million.

2.2.3.2 In the absence of credit risk, the contract cash flows are simple to specify. For simplicity we will refer to 31 December 2004 as time $t = 0$, with $t = 1$ representing 31 December 2005 etc. Now let $m(y, x)$ represent the crude central death rate for age x published by the Office for National

Statistics in the year y . We then construct a survivor index $S(t)$ as follows:

$$S(0) = 1$$

$$S(1) = S(0) \times (1 - m(2003, 65))$$

$$S(t) = S(0) \times (1 - m(2003, 65)) \times (1 - m(2004, 66)) \times \dots \times (1 - m(2002 + t, 64 + t)).$$

At each time $t = 1, 2, \dots, 25$, the bond pays a coupon of £50 million $\times S(t)$.

2.2.3.3 These cash flows are illustrated in Figure 3. As far as investors are concerned, they make an initial payment of around £540 million (i.e. the issue price) and receive in return an annual mortality-dependent payment of £50 million $\times S(t)$ in each year t for 25 years.

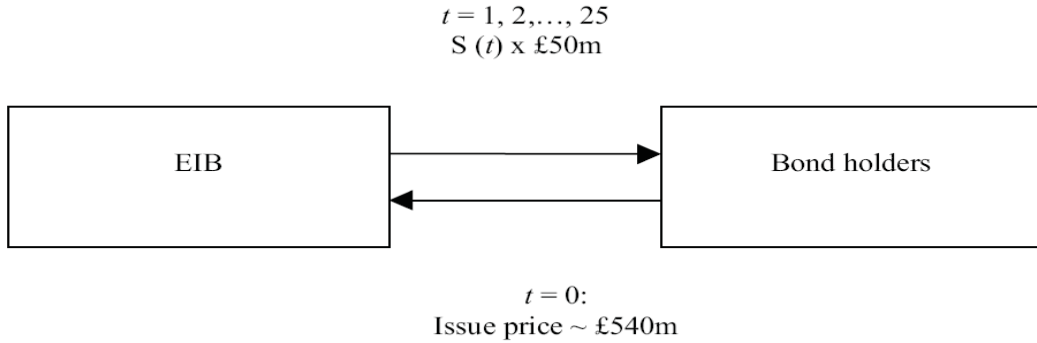


Figure 5: Cash flows from the EIB/BNP Bond, as viewed by investors. Source: Blake *et al.*, 2006

2.2.3.4 Although the bond was never launched, the issue price was determined by BNP Paribas as follows:

- i) Ignoring for the moment the £50 million multiplier, the contract specifies a set of anticipated cashflows $S(t)$ based on the Government Actuary's Department's 2002-based projections of mortality.
- ii) Each projected cashflow is priced by discounting at LIBOR minus 35 basis points. The EIB curve typically stands about 15 basis points

below the LIBOR curve, so that investors in the longevity bond are being asked to pay 20 basis points to hedge their longevity risk. For further discussion of this risk premium, the reader is referred to Cairns, Blake, Dawson & Dowd (2005) and Cairns, Blake & Dowd (2005).

2.2.3.5 The details given above describe the cash flows from the point of view of the investors. However, there are also issues of credit risk to consider, and these lead to some complex background details. These details and the involvement of BNP Paribas and Partner Re are represented in Figure 4.

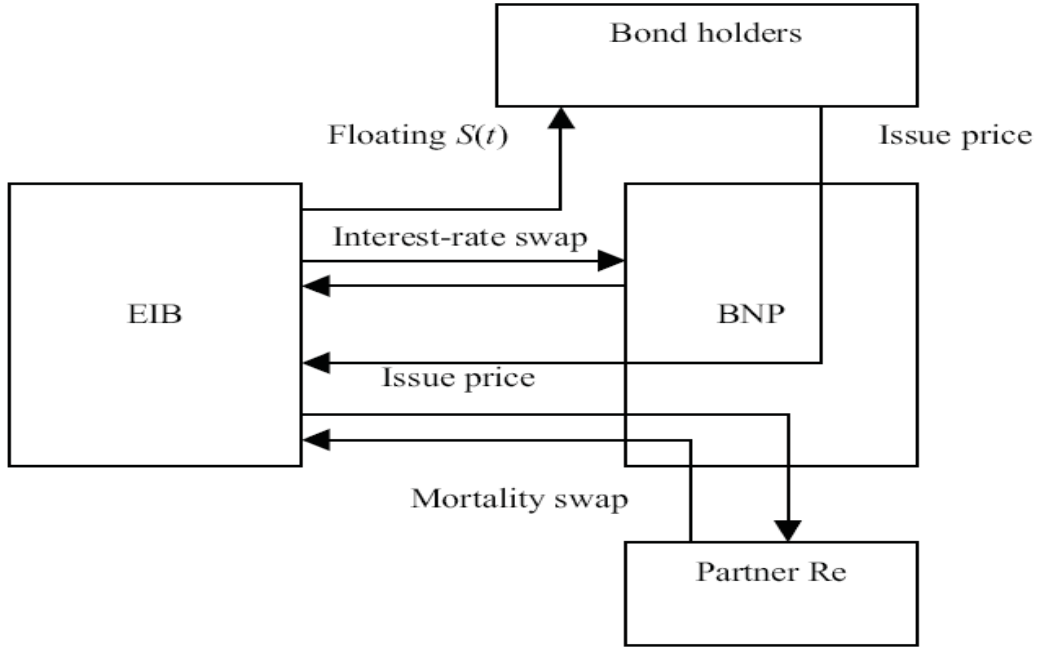


Figure 6: Cash flows from the EIB/BNP bond. Source: Blake *et al.* (2006)

The longevity bond is actually made up of 3 components. The first is a floating rate annuity bond issued by the EIB with a commitment to pay in euros (€). The second is a cross-currency interest-rate swap between the EIB and BNP Paribas, in which the EIB pays floating euros and receives fixed sterling. These fixed payments, $\hat{S}(t)$, might be, but do not have to be, equal to the $\bar{S}(t)$. (The fixed rate, $\hat{S}(t)$, has to be set to ensure that the swap has zero value at initiation. Typically, this would require the fixed rate

to be close but not equal to $\bar{S}(t)$. From the EIB's perspective, this converts the first element, the floating-rate bond, into a fixed-rate £ bond. The third and most distinctive component is a mortality swap between the EIB and Partner Re, in which the EIB exchanges the fixed sterling $\hat{S}(t)$ for the floating sterling $S(t)$ at each of the payment dates, $t = 1, 2, \dots, 25$. Strictly speaking, the third component is an OTC deal between BNP and Partner Re. The second component then becomes a commitment from BNP to pay $\text{£}S(t)$ to the EIB, rather than $\text{£}\bar{S}(t)$, in return for floating €. For this reason, we see in Figure 6 that the mortality-swap cash flows are directed through BNP. Ignoring credit risk, the result of the two swaps from the perspective of the EIB is to convert floating € into $\text{£}\bar{S}(t)$. The intermediate swap of floating € for floating $\text{£}\hat{S}(t)$ does not (as noted above) require that $\hat{S}(t) = \bar{S}(t)$: the price agreed for this swap will, however, depend on what level the $\hat{S}(t)$ are set at. Similarly the price for the mortality swap will depend on the $\hat{S}(t)$.

2.2.3.6 Note that the second component implies that EIB and BNP have potential credit exposures to each other, and such exposures would become manifest if underlying random factors change and the swap value moves away from 0 (in which case the swap would become an asset to one party and a liability to the other). The third component implies that BNP has a credit exposure to Partner Re. The parties concerned might (or might not) wish to take out some form of insurance on these various credit exposures.

2.2.3.7 It is important to appreciate what is going on here in plain language. In a nutshell, the bond is issued by the EIB, and investors only face a credit exposure to the EIB. The EIB has a commitment to make mortality-linked payments in sterling, and then engages in a swap with BNP to exchange this commitment for a commitment to make floating euro payments. In entering into this swap, BNP takes on mortality exposure, which it then hedges with Partner Re. Thus, if Partner Re defaults, that is BNP's problem, and if BNP defaults, that is the EIB's problem. However, EIB is still committed to pay investors regardless of whether Partner Re or BNP default or not.

2.2.3.8 For their part, investors have the protection of the EIB's commitment to repay, backed by the EIB's AAA credit rating. For its part, the EIB

has the protection of BNP's commitment to take on the bond's longevity risk exposure, and this commitment is backed by BNP's AA credit rating and by the knowledge that BNP has reinsured that risk with Partner Re. For its part, BNP has the protection of the reinsurance provided by Partner Re, whose rating is also AA.

2.2.3.9 The EIB/BNP longevity bond has some attractive features:

- 1.) Its cash flows are designed to help pension plans hedge their exposure to longevity risk. To be more precise, they provide an ideal hedge against a notional annuity provider who is committed to providing level annuity payments to the reference population over a horizon of 25 years.
- 2.) The survivor index $S(t)$ is calculated with reference to crude death rates published by the ONS¹³. These death rates are a reliable and easily obtainable public source. This helps reassure investors that they would have full access to the data and would not lose out as a result of insurance companies manipulating their reported death rates. The use of crude death rates also avoids arguments over smoothing methodologies.
- 3.) Trends in national mortality should provide a reasonable match for trends in annuitants' mortality, and thus help to reduce basis risk in an annuity book that might be hedged by an investment in the longevity bond.

2.2.3.0 As noted earlier, the EIB/BNP longevity bond was only partially subscribed and was later withdrawn for redesign. There seem to be a number of reasons for its slow take up and perhaps lessons can be learned for future contract design:

- 1.) It is likely that a bond with a 25 year horizon provides a less effective hedge than a bond with a longer horizon. (Evidence to this effect is provided by Dowd, Cairns & Blake, 2005.) Similarly, the bond might prove to be a less effective hedge for pension liabilities linked to different

¹³Official national statistics department

age cohorts or to females. This means that the EIB bond might not be a particularly effective hedge for the kind of annuity book for which it was designed, and this consideration might have discouraged annuity providers from investing in it.

- 2.) The amount of capital required is high relative to the reduction in risk exposure. This makes the BNP bond capital-expensive as a risk management tool.
- 3.) The degree of model and parameter risk is quite high for a bond of this duration (see, for example, Cairns, Blake & Dowd, 2005), and this degree of uncertainty might make potential investors and issuers uncomfortable. Thus, even if the bond provides a perfect hedge, there will be uncertainty over what the right price to pay or charge should be.
- 4.) Potential hedgers might feel that the level of basis risk is too high relative to the price being charged. For example, basis risk can arise because annuitants are likely to experience more rapid mortality improvements than is reflected in the population-wide index on which the payments are determined. Basis risk can also arise because the longevity bond specifies level annuity payments, whereas most real-world pension schemes allow for escalating (i.e. inflation-linked) payments. A further cause for basis risk is inaccuracy in the estimates of number of deaths (e.g. people dying while on holiday, slow notification of pensioner death) or in the number exposed to risk (e.g. the number exposed to risk is based on population projections from the last census date), or a failure to ensure these correspond.
- 5.) The underlying index is calculated with reference to public mortality death rates. However, the use of public death rates means that $S(t)$ will underestimate the true proportion of the cohort that survive. A more natural definition for the survivor index, which avoids this bias, would make reference to mortality rates: that is, $S(t) = S(0) \times (1 -$

$q(2003, 65)) \times (1 - q(2004, 66) \times \dots \times (1 - q(2002 + t, 64 + t))$ where the $q(y, x)$ are mortality rates for age x in year y ."

2.3 Recent Developments

If a new market evolves Investment banks need not much time to create financial instruments to participate. Usually Investmentbanks offer different variations of derivatives to hedge or just to speculate. Derivatives means that the value of a financial instrument is derived out of an underlying, event or condition. In our case a longevity or mortality index. Longevity and mortality derivatives are hence no contracts of insurance, they are capital market instruments with payoffs linked to the value of an pre-defined underlying index.

Biffis and Blake (2009) argue that mortality and longevity swaps attract the greatest attention from insurers and investment banks. For instance the EIB bond as described in Section 2.2.3 has also a derivative component, i.e. a longevity swap, since fixed payments from investors in the bond were intended to be swapped for coupons linked to the annual number of survivors in the relevant cohort. In April 2007 SwissRe agreed to act as the floating rate payer, i.e. the risk taker per contra to the fixed rate payer, in a swap contract with Friends Provident, a UK life insurer, in exchange for an undisclosed premium, see Biffis and Blake(2009). The £1.7 billion contract was the first which was publicly announced Bowe et al. (2006) describes two common reinsurance models, the longevity swap and the quote-share-reinsurance. Whereas the quote share reinsurance is just a longevity swap plus a hedge against the investment risk. In 2.3.1 and 2.3.2 the swaps are explained, see Bowe et al. (2006).

2.3.1 The longevity swap

The longevity swap is especially developed for closed annuity portfolios in the annuity – reference – time. Simplified could be stated, that a predicted or expected cash flow of annuities is swapped against a actual cash flow of annuities, i.e. a Swap. The primary insurer ceded a well defined proportion of his running annuities to the reinsurer. He pays a yearly or during the period reinsurance premium, defined before the inception of treaty. This premium equals exactly the expected annuity payments of the single years ,

calculated on the basis of a jointly agreed mortality table. The underlying mortality table of the reinsurance contract is fitted to the reinsured portfolio. The reinsurance premium of the cedent include the annuity payment and the expense loadings of reinsurer. Are they once fixed, the reinsurance premiums, i.e. the expected annuity payments, will be unaltered until the expiry of the treaty. Hence the cedent have a plan reliability over the reinsured portfolio. In return the cedent gets as service from the reinsurer all actual annuity payments until the last reinsured policy is expired. Finally the primary insurer has only to bear the investment risk for the reinsured portfolio, whereas the longevity risk is completely swapped to the reinsurer. Figure 7 should illustrate the procedure.

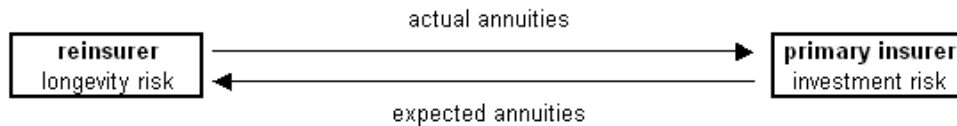


Figure 7: Diagram of the longevity swap. Source: Analyse und Absicherung der Risiken im Lebensversicherungsgeschäft, Bowé et.al., 2006

Alongside guaranteed annuities which are paid since the beginning of the reinsurance, contractual increases of annuities could also be hedged against a suitable premium.

2.3.2 The quote – share – reinsurance

The quote – share – reinsurance is nothing else than a longevity swap plus a hedge against the investment risk. Obviously a third party is required to bear the investment risk, for instance a bank or a other financial institution. The third party gets a single premium from the cedent to finance the investment risk and to pay the expected annuity to the reinsurer. The reinsurer pays in turn the actual annuity to the cedent. Unlike to the longevity swap, the primary insurer gets for a single premium and not for a running premium, the actual annuities. To realize this modified longevity swap a “special – purpose – vehicle” (SPV) has to be founded. This action is comparable to a

securitization because there is also a SPV required. The purpose of the SPV is the hedge of the contract partner against a default of other contract parties. Obviously the costs increase with complexity of the reinsurance construction. Therefore is this extended version of a longevity swap only adequate for huge portfolios with running annuities. Figure 8 shows the corresponding scheme.

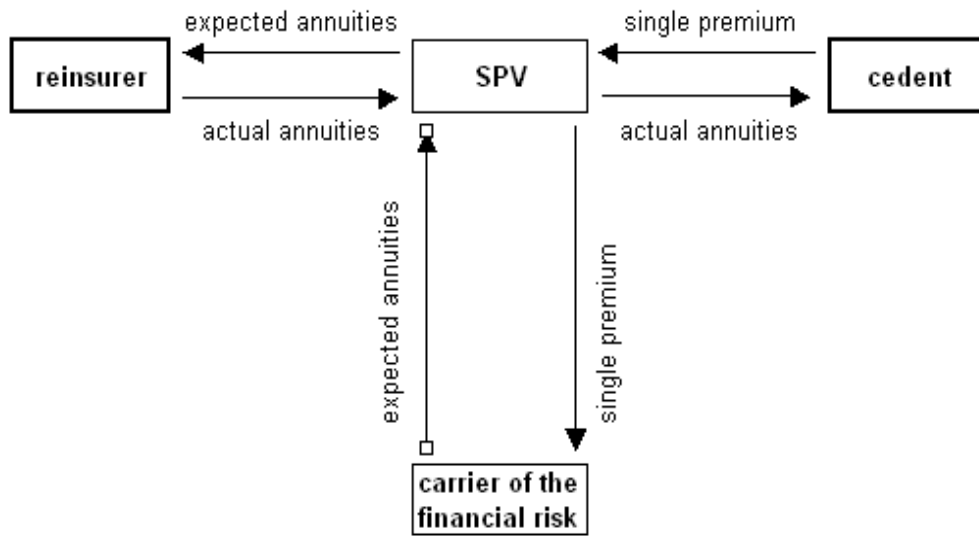


Figure 8: Diagram of the quote - share - reinsurance. Source: Analyse und Absicherung der Risiken im Lebensversicherungsgeschäft, Bowé et.al., 2006

The longevity swap is a transparent reinsurance model, which allows a clear distinction between the financial – and the longevity risk and it offers the primary insurer the possibility to cede the single risks at a adequate premium to the co - contractor.

2.3.3 JP Morgans *q-forward*

Biffis and Blake (2009) states that a swap could be synthesized by combining together several mortality forwards. This kind of contracts have been marketed by JP Morgan since July 2007, under the name of *q-forwards* (see Coughlan et al.(2007) and Loeys et al. (2007)). To relieve the understanding of a *q-forward*, the idea behind a common forward contract should be

explained shortly. Forward contracts have seldom standards and are most of the time traded “Over the Counter” (OTC), i.e. not traded on regulated exchanges. The principle is to “lock-in” a price for a commodity, exchange rate or similar goods at the initial trade date. It is a obligation to buy or sell a financial instrument or to make a payment at some point in the future, the details of which were settled privately between the two counterparties. Forward contracts generally are arranged to have zero mark-to-market value at inception. The mark-to-market accounting is nothing else than a revaluation of a financial instrument at regular intervals. Examples include forward foreign exchange contracts in which one party is obligated to buy foreign exchange from another party at a fixed rate for delivery on a pre-set date.

Coughlan et al. (2007) a member of JP Morgans Pension Advisory Group introduces the *q-forward* as a agreement between two parties to exchange at a future date (the maturity of the contract) an amount proportional to the realized mortality rate of a given population (or subpopulation), in return for an amount proportional to a fixed mortality rate that has been mutually agreed at inception. In other words, a *q-forward* is a zerocoupon swap that exchanges fixed mortality for realized mortality at maturity. This is illustrated in Figure 9. The reference rate for settling the contract is the realized mortality rate as determined by the appropriate index, such as the LifeMetrics Index.

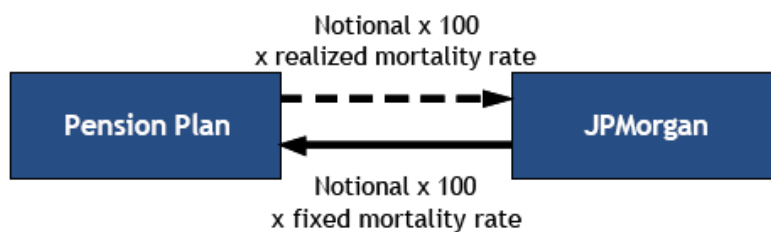


Figure 9: A *q-forward* contract to hedge the longevity risk of a pension plan (or an annuity book). Source: Coughlan et al., (2007)

In a fair market, the fixed mortality rate at which the transaction takes place defines the "forward mortality rate" for the population (or subpopulation) in question. If the *q-forward* is fairly priced, no payment changes hands

at the inception of the trade. At maturity, however, a net payment will be made by one counterparty or the other. Figure 10 gives an example term sheet for a *q-forward* transaction, where the reference population corresponds to 65-year-old males in England & Wales.

Notional Amount	GBP 50,000,000
Trade Date	31 Dec 2006
Effective Date	31 Dec 2006
Maturity Date	31 Dec 2016
Reference year	2015
Fixed Rate	1.2000 %
Fixed Amount Payer	JPMorgan
Fixed Amount	Notional Amount x Fixed Rate x 100
Reference Rate	LifeMetrics graduated initial mortality rate for 65-year-old males in the reference year for England & Wales national population Bloomberg ticker: LMQMEW65 Index <GO>
Floating Amount Payer	XYZ Pension
Floating Amount	Notional Amount x Reference Rate x 100
Settlement	Net settlement = Fixed amount – Floating amount

Figure 10: An illustrative term sheet for a single *q-forward* to hedge longevity risk. Source: Coughlan et al., 2007

The *q-forward* payout is determined by the value of the LifeMetrics Index for this subpopulation at the maturity of the contract. This transaction is a 10-year *q-forward* contract initiated on 31 December 2006 and maturing on 31 December 2016. It reflects part of a longevity hedge provided to a UK pension plan. At maturity the hedge provider (the fixed-rate payer) pays to the pension plan an amount proportional to a fixed mortality rate of 1.2000%. In return the pension plan pays to the hedge provider an amount determined by the reference rate at maturity, which corresponds to the most

recent value of the LifeMetrics Index reflecting the realized mortality rate for 65-year-old males in England & Wales. Because of the ten-month lag in the availability of official data, settlement on 31 December 2016 will be based on the LifeMetrics Index level for the reference year 2015.

The settlement that takes place at maturity is based on the net amount payable and is proportional to the difference between the fixed mortality rate (the transacted forward rate) and the realized reference rate. Figure 14 shows the settlement calculation for different potential outcomes for the realized reference rate. If the reference rate in the reference year is below the fixed rate (i.e., lower mortality) then the settlement is positive, and the pension plan receives the settlement payment to offset the increase in its liability value. If, on the other hand, the reference rate is above the fixed rate (i.e., higher mortality) then the settlement is negative and the pension plan pays the settlement payment to the hedge provider, which will be offset by the fall in the value of its liabilities.

Reference Rate (Realized Rate)	Fixed Rate	Notional	Settlement
1.0000 %	1.2000 %	50,000,000	10,000,000
1.1000 %	1.2000 %	50,000,000	5,000,000
1.2000 %	1.2000 %	50,000,000	0
1.3000 %	1.2000 %	50,000,000	-5,000,000

Figure 11: An illustration of q-forward settlement for various outcomes of the realized reference rate. A positive(negative) settlement means the fixed-rate receiver receives (pays) the net settlement amount. Source: Coughlan et al., 2007

Concerning the pricing, Loeys (2007) states that there are more agents in the economy who are short longevity (i.e., are financially hurt by unexpected rises in longevity) than those who are long. The market is thus net short longevity. To transfer this risk, it needs to attract investors who require compensation to take on this risk. A pension fund that hedges its longevity risk expects to be paid by the investors if mortality falls by more than expected and is willing to pay if mortality ends up higher, because its own

cash outflows will then be less. As a result, the longevity forward that will attract investors into this market must lie *above* the expected mortality rate, see Figure 12. This discount, therefore, constitutes the expected return to the investors of taking on mortality risk. And this return needs to provide a sufficient return to risk to be competitive with other assets the investor could buy. To adept more about the pricing of a *q-forward* contract the interested reader is referred to chapter 6 and further to "Longevity: a market in the making", Loeys et al. (2007) or Bauer et al. (2008) who gives a more general overview on the pricing of mortality linked securities. However a simulation of a hedge through a swap is presented in chapter 6.

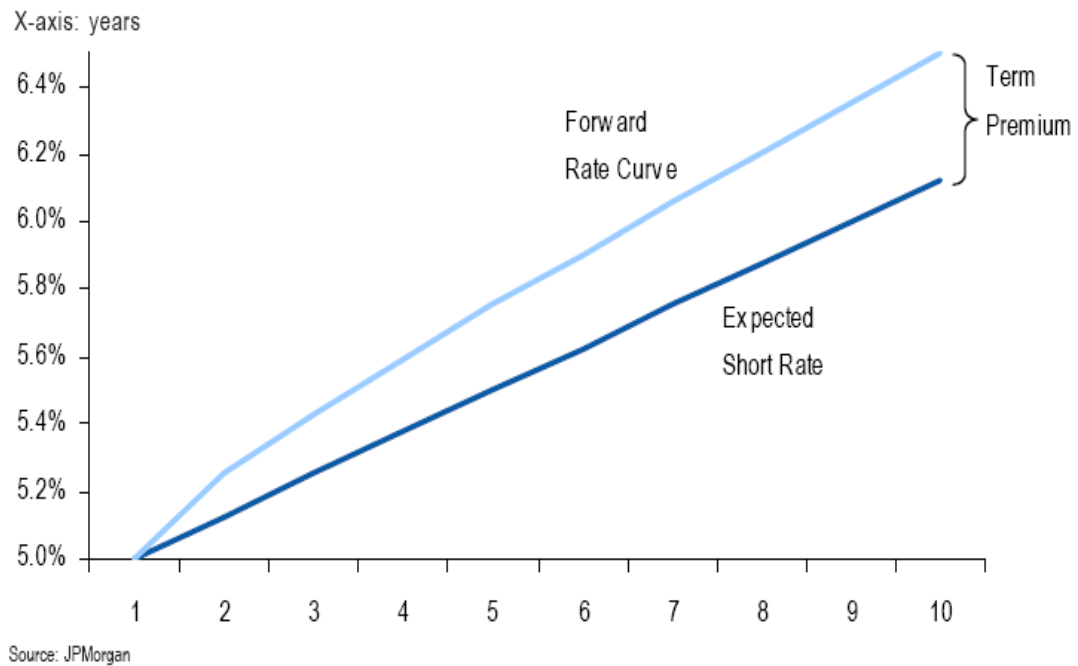


Figure 12: Term Premium, Forward Rate and Expected Short Rate of a *q-forward*. Source: Loeys, et al. (2007): *Longevity: a market i the making*;JP-Morgan

The first hedge of longevity risk takes place at January 2008, between Lucida plc and JP Morgan. Lucida plc, a new insurance company involved in the pension buyout market, formed to take on longevity risk and corporate pension schemes, recently announced a deal with JPMorgan to hedge longevity

risk through a derivative contract linked to the LifeMetrics Longevity Index. The contract, the first of its kind involving an Insurer, signals continued progress in the development of what many believe to be a significant new market. (see lucidapl.com, press release).

2.3.4 Longevity futures and options

Blake et al. (2006) gives an extensive description of the possible constructions concerning the futures and options market. Biffis and Blake (2009) argues that at current time no futures or options markets on mortality linked securities are active to date. However considerable effort is being spent by reinsurers and investment banks to explore opportunities for innovation. Natixis, for example, has launched a longevity-driven collar¹⁴.

¹⁴Combination of options, i.e. call or put

3 Longevity Indices

In order to enter into a contract based on a standardized portfolio of lives, a suitable longevity index is needed. Sweeting (2010) distills in his paper 13 criteria for longevity indices out of the foregone work of Bailey (1992):"

- unambiguous – the reference population on which the indices are based should be defined in detail, including details of how individuals can enter and leave the index (other than through death);
- transparent – the methods used to graduate mortality rates should be clear;
- objective – graduation methods should have as little subjective input as possible;
- measurable – the mortality experience of the reference population should be capable of being measured;
- timely – the mortality experience of the reference population should be available shortly after the effective date of that experience;
- regular – the indices should be produced in accordance with a pre-arranged timetable;
- appropriate – the indices should reflect the composition of the populations requiring hedging;
- popular – the indices should be few enough that securities, derivatives and swaps based on them will be liquid;
- relevant – the variability of the liabilities being hedged relative to the indices should be significantly lower than their volatility relative to population longevity;
- highly correlated – the correlation between $Li - Lt$ and $Lp - Lt$ should be strongly positive, where Lt is the value of the floating leg a longevity swap based on the total population, Li is the value of the floating leg

of the more specific index-based longevity swap, and Lp is the value of a pension scheme's liabilities;

- reflective of current hedging needs – the structures of the indices should reflect the needs of those using them to hedge;
- stable – the criteria used to construct indices should change only infrequently; and
- specified in advance – the indices should be defined in advance as far as possible, and there should be an independent committee to deal with issues when this is not possible."

3.1 Existing Indices

Longevity indices calculated out from mortality tables, whose main purpose is to value actuarial liabilities or mortality linked securities and derivatives do exist. The indices launched are discussed below:

3.1.1 Life Metrics

JP Morgan provides longevity indices for the United States, Germany, the Netherlands, and England and Wales in its LifeMetrics suite, developed with the assistance of the Pensions Institute at Cass Business School and Watson Wyatt. For all of the regions mentioned, rates are based on mortality of the entire population. For England and Wales, LifeMetrics takes raw data from the UK Statistics Authority (formerly the Office for National Statistics) and applies a pre-defined smoothing algorithm. This index fulfils many of the criteria described above. However, since the population used is the entire England and Wales population, the relevance of this data to a specific group of lives such as a pension scheme or a book of annuitants is questionable, see Sweeting(2010) and the technical document of the LifeMetrics indices.¹⁵.

¹⁵http://www.jpmorgan.com/cm/cs?pagename=JPM/DirectDoc&urlname=lifemetrics_technical.pdf

3.1.2 Xpect

Xpect from Deutsche Börse AG produces monthly mortality data for Germany. How the data for the monthly calculation of rates is collected is not explained, see the factsheet of the Xpect Indices¹⁶.

3.1.3 QxX

Finally, Goldman Sachs developed the QxX index, which it offered until December 2009. This was a longevity index covering medically underwritten US lives. This had the advantage of being objectively calculated, as with the indices above, and was calculated more frequently (on a monthly basis). The number of lives covered was small (46,290 at outset), and the class of business was not necessarily relevant for the hedging of pensions (it covered the life settlements market), though it was perhaps more appropriate than an index based on a national population. However, as indicated above, Goldman Sachs decided in December 2009 to wind down its life settlements index, see Sweeting (2010).

Longevity indices could serve a useful role in facilitating the hedging of longevity risk in pension schemes. The characteristics of good indices are numerous, and whilst the criteria for good benchmarks as discussed by Sweeting (2010) are useful, the nature of longevity indices means that additional considerations are needed. In particular, the scope for subjectivity and the construction of the indices could have a major impact on the success of indices. Longevity indices do exist, but since they are based largely on national population data, their relevance is perhaps limited. This suggests that new, more focussed indices would be a useful addition for those wishing to hedge longevity. Good hedging results can be achieved using a relatively small number of swap contracts at key combinations of age and term. Such an approach would help to develop a liquid market in such swaps, see Sweeting (2010).

¹⁶<http://www.xpect-index.com/files/pdf/Factsheet%20Xpect%20Data%20e.pdf>

4 Empirical Part

4.1 Stylized facts on longevity and mortality

To value the underlying risk and the expected future profits of financial instruments which are linked to mortality rates it is necessary to know some basic facts about past changes in longevity. For that reason the reader should be versant with the basic terminology of mortality rate measures, if not they are explained in Appendix A. To figure out stylized facts, Austrian Mortalitytables was used¹⁷. (for US and UK see Loeys et al., 2007):

- i) Mortality rises with age. This is not surprising, the older you are the higher the probability you will die in a given year. The expected residual lifetime e_x , i.e. the number of years you are still expected to live, falls with age. Further the mortality rate q_x rises approximately exponentially with age. However, one can see in Figure 13 the logarithmic mortality rates. The variance increases with respect to high ages due to the lack of data from the age of 95 on.

¹⁷Source: MHD - Mortality Human Database

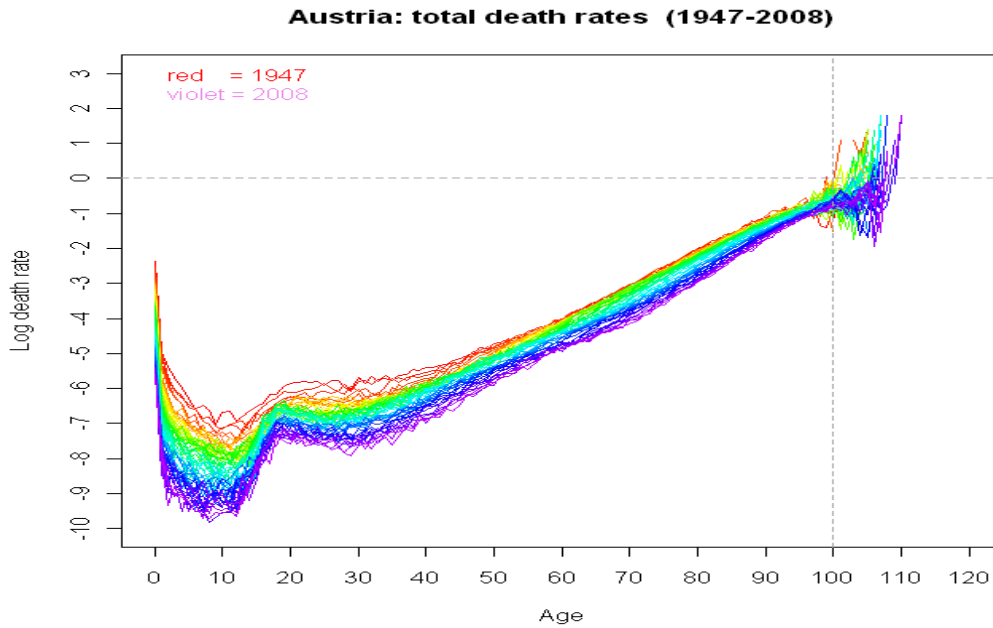


Figure 13: Austrian total logarithmic mortality rates 1947(upper) - 2008(lower), own calculation. Data: MHD

- ii) The life expectancy of females is higher, but the gap becomes narrower over the years. In 1950 the q_x for 60 year old Austrian males was 2,22% and for females 1,36%, hence the difference was 0,86 Percentage Points. In 2008 the rates for males improved to 1,06% and for females to 0,48% that's a gap of 0,58 Percentage Points, see Figure 14.

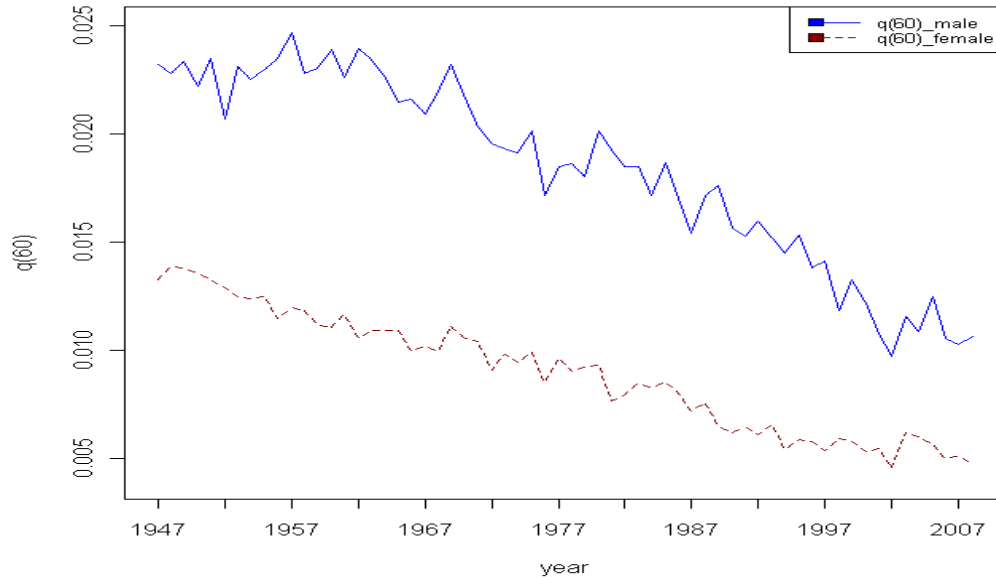


Figure 14: Austrian male and female mortality rates (1947-2008). Data: Statistik Austria

- iii) The changes in mortality rates have been quite volatile over time. The volatility of the YoY percentage change of the mortalityrate q_x of 60 year old Austrian males from 1947 to 2008 was 7,59%,see Fig.15. Further the trend line is descending, what implies a decreasing mortality over time.

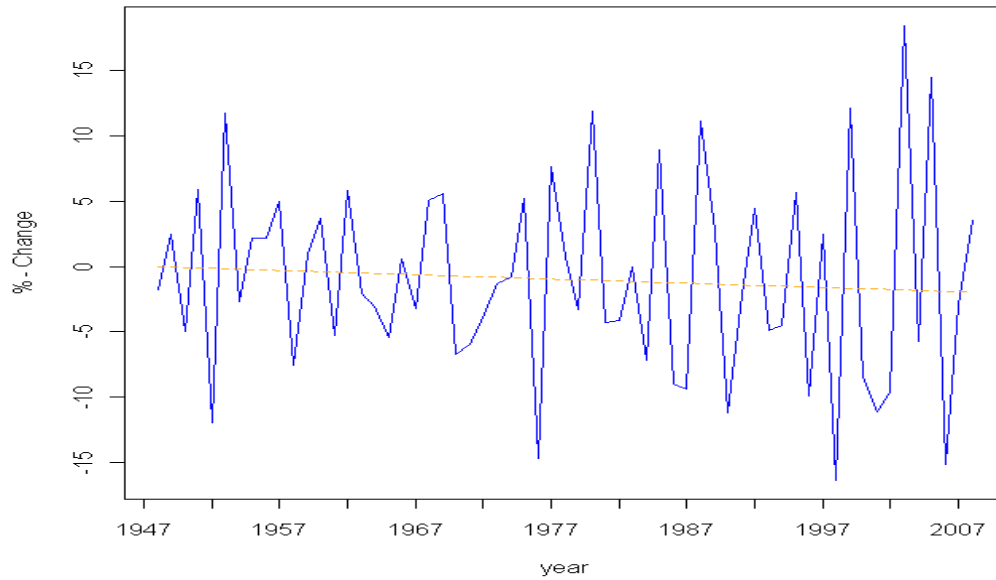


Figure 15: % - change of mortality rates, with trendline, of 60 year old Austrian male, own calculation. Data: Statistik Austria

Overall one can see that this stylized facts confirm the efforts taken to create a new market. The upward trend of the survival probability with respect to time should suffice a hedge anyway.

4.2 Forecasting and sampling of mortality rates

The demographic change caused much effort to develop forecasting models for mortality rates during the last two decades. Reviews and comparisons are given in Cairns et al. (2008) and Booth et al. (2006). Booth et al. describes the Lee-Carter Model (Lee and Carter 1992) as a milestone in forecasting mortality rates. Lee and Carter modeled the yearly American mortality rates from 1900 - 1987 as

$$\ln({}_tq_x) = a_x + b_x k_t + \epsilon_{t,x} \text{ or } {}_tq_x = e^{a_x + b_x k_t + \epsilon_{t,x}} \quad (2)$$

Essentially Lee and Carter describes the logarithmically transformed age-specific mortality rate ${}_tq_x$ at age x in year t , as the sum of an age specific component that is independent of time a_x , and the product of a time varying parameter k_t (also known as the mortality index) that summarizes the general level of mortality and an additional age-specific component b_x which describes the way mortality varies at the age of x as a reaction to the change of the level of the mortality index k_t . The final term $\epsilon_{t,x}$ is the residual at age x and time t .

Constraints are imposed to obtain a unique solution:

1. the a_x are set equal to the arithmetic means over time of $\ln({}_tq_x)$
2. the b_x sum to *unity*
3. the k_t sum to *zero*.

As all parameters of the right hand side of the equation are unobservable fitting the model by ordinary least squares (OLS) is not possible. To handle this problem, Lee and Carter(1992), used a two stage estimation procedure to compute the parameters. Thus they first applied singular value decomposition (SVD) to the matrix of

$$[\ln({}_tq_x) - a_x] \quad (3)$$

to obtain estimates of b_x and k_t . In the second stage estimation, the a_x and b_x from the first step are taken as given and the time series of k_t is reestimated by solving for k_t such that

$$D(t) = \sum_x [N(x, t) e^{a_x + b_x k_t + \epsilon_{t,x}}], \quad (4)$$

where $D(t)$ is the total number of deaths in time t , for a given population age distribution $N(x, t)$. Thereby a new estimate of k_t was found, such that for each year, given the actual population age distribution, the implied number of deaths will equal the actual number of deaths. This is to ensure that the mortality schedules fitted over the sample years will reconcile the total number of deaths and the population age distributions. Further an autoregressive integrated moving-average (ARIMA) model is used to model the dynamics of k_t . The Box and Jenkins (1976) approach often is employed to obtain a fitted ARIMA model from the empirical k_t data. It usually proved to be adequate a random walk with drift $ARIMA(0, 1, 0)$, see Lee and Carter (1992), Lazar (2004) and Li and Chan (2005).

4.3 The forecasting model

To forecast and sample mortality rates I followed the work from Hyndman and Ullah (2006) - "Robust forecasting of mortality and fertility rates: A functional data approach". Whereas the first author provides an R-package which implements the methodology. The package is called "*demography*" and could be downloaded on the authors homepage¹⁸.

4.3.1 What is Functional Data

Functional domain supports many recent methodologies for statistical analysis of data coming from measurements concerning continuous phenomena;

¹⁸<http://robjhyndman.com/>

such techniques constitute nowadays a new branch of statistics named functional data analysis, see Ramsay and Silverman (1997, 2002). Functional data are essentially curves and trajectories. The basic rationale is that we should think of observed data functions as single entities rather than merely a sequence of individual observations. Even though functional data analysis often deals with temporal data, its scope and objectives are quite different from time series analysis. While time series analysis focuses mainly on modeling data, or in predicting future observations, the techniques developed in FDA are essentially exploratory in nature: the emphasis is on trajectories and shapes; moreover unequally-spaced and/or different number of observations can be taken into account as well as series of observations with missing values. From a practical point of view, functional data are usually observed and recorded discretely. Let $\{\omega_1, \dots, \omega_n\}$ be a set of n units and let $y_i = (y_i(t_1), \dots, y_i(t_p))$ be a sample of measurements of a variable Y taken at p times $t_1, \dots, t_p \in T = [a, b]$ in the i -th unit ω_i , ($i = 1, \dots, n$). As remarked above, such data y_i ($i = 1, \dots, n$) are regarded as *functional* because they are considered as single entities rather than merely sequences of individual observations, so they are called *raw functional data*; indeed the term functional refers to the intrinsic structure of the data rather than to their explicit form. In order to convert raw functional data into a suitable functional form, a smooth function $x_i(t)$ is assumed to lie behind y_i which is referred to as the *true functional form*; this implies, in principle, that we can evaluate x at any point $t \in T$. The set $\chi_T = \{x_1(t), \dots, x_n(t)\}_{t \in T}$ is the *functional dataset*, see Ingrassia and Costanzo (2005).

4.3.2 Functional Principal Component Analysis

Principal component analysis (PCA) is a standard approach to the exploration of variability in multivariate data. PCA uses an eigenvalue decomposition of the variance matrix of the data to find directions in the observations space along which the data have the highest variability. For each principal component, the analysis yields a loading vector or weight vector which gives the direction of variability corresponding to that component. In the func-

tional context, each principal component is specified by a principal component weight function or *eigenfunction* $\xi(t)$ defined of the same range of t as the functional data. The aim of PCA is to find the weight function $\xi_j(t)$ that maximizes the variance function of the principal component scores z_i , i.e. the orthogonal decomposition of the variance function, see Daniele(2006):

$$v(s, t) = \frac{1}{n-1} \sum_{i=1}^n \{z_i(s) - \bar{z}(s)\} \{z_i(t) - \bar{z}(t)\}, \quad (5)$$

(which is the counterpart of the covariance matrix of a multidimensional dataset) in order to isolate the dominant components of functional variation, see e.g. also Pezzulli (1994). In analogy with the multivariate case, the functional PCA problem is characterized by the following decomposition of the variance function:

$$v(s, t) = \sum_j \lambda_j \xi_j(s) \xi_j(t) \quad (6)$$

where λ_j, ξ_j satisfy the eigenequation:

$$\langle v(s, \cdot), \xi_j \rangle_h = \lambda_j \xi_j(t). \quad (7)$$

and the eigenvalues:

$$\lambda_j := \int_T \xi_j(s) v(s, t) \xi_j(t) ds dt \quad (8)$$

are positive and non decreasing while the eigenfunctions must satisfy the constraints:

$$\begin{aligned} \int_T \xi_j^2(t) dt &= 1 \\ \int_T \xi_j(t) \xi_i(t) dt &= 0 \quad \forall i, j \quad i < j \end{aligned} \quad (9)$$

The ξ_j are usually called *principal component weight functions*. Finally the principal component scores (of $\xi(t)$) of the units in the dataset are the values ω_i given

by:

$$w_i^{(j)} := \langle z_i, \xi_j \rangle = \int_T \xi(t) z_i(t) dt. \quad (10)$$

The decomposition (7) defined by the eigenequation (8) permits a reduced rank least squares approximation to the covariance function v . Thus, the leading eigenfunctions define the principal components of variation among the sample functions z_i , see Ingrassia and Costanzo (2005) and Daniele (2006).

The principal components analysis of functional data is often enhanced by the use of smoothing, see Silverman (1996). To obtain a smoothed functional PCA, we have to control the size of ξ , but also its roughness. In practice, the constraints are replaced by the following ones

$$\int \xi^2(t) dt + \lambda \int \{\xi''(t)\}^2 dt = 1 \quad (11)$$

$$\int \xi_j(t) \xi_i(t) dt + \lambda \int \xi_i''(t) \xi_j''(t) dt = 0 \quad \forall i, j \quad i \neq j. \quad (12)$$

The smoothing parameter $\lambda \geq 0$ controls the amount of smoothing inherent in the procedure. The smoothing parameter choice is usually a consequence of empirical subjective considerations together with a cross-validation criterion, see Daniele (2006).

4.3.3 The model used

Hyndman and Ullah show in their article that the MSE of their model is superior to four different models as shown in Figure 16.

The superior performance of this approach arise for the following reasons: (1) They allow more complex dynamics than other methods by setting $K > 1$ (Lee and Carter (1992) $K = 1$), thus allowing more than one principal component; (2) nonparametric smoothing reduces the observational noise; (3) the use of robust methods avoids problems of outlying years, i.e. flu pandemic, world wars. Further, it has added advantage of providing interesting

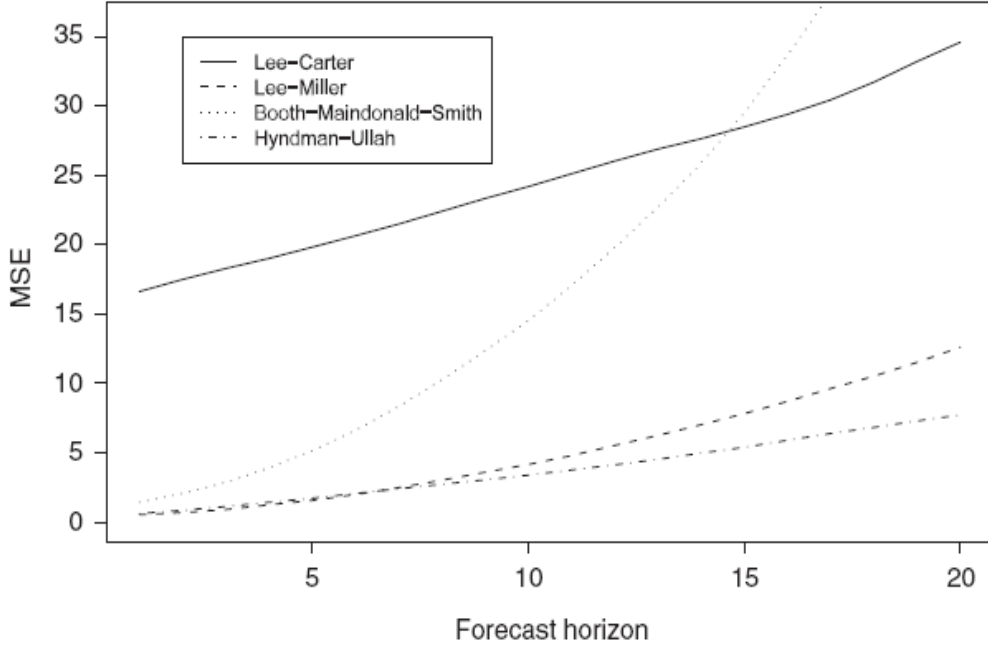


Figure 16: $m = 1959, \dots, 2000$. Forecast period: $m + 1, \dots, \min(2001, m + 20)$. Source: Hyndman and Ullah (2006)

historical interpretation of dynamic changes by separating out the effects of several orthogonal components.

They define the model in the following way. Let $y_t(x)$ denote the log of the observed mortality or fertility rate for age x in year t . We assume there is an underlying smooth function $f_t(x)$ that we are observing with error and at discrete (and possibly sparse) points of x . Our observations are $\{x_i, y_t(x_i)\}$, $t = 1, \dots, n, i = 1, \dots, p$ where

$$y_t(x_i) = f_t(x_i) + \sigma_t(x_i)\varepsilon_{t,i}, \quad (13)$$

$\varepsilon_{t,i}$ is an iid standard normal random variable and $\sigma_t(x_i)$ allows the amount of noise to vary with x . Typically x_1, \dots, x_p are single years of age ($x_1 = 0, x_2 = 1, \dots$) or denote 5-year age groups. We are interested in forecasting $y_t(x)$ for $x \in [x_1, x_p]$ and $t = n + 1, \dots, n + h$. Note that the data are

not directly of a functional nature, but that we assume there are underlying functional time series which we are observing with error at discrete points.

The final approach of Hyndman and Ullah is summarized below.

- (1) Smooth the data for each t using a nonparametric smoothing method to estimate $f_t(x)$ for $x \in [x_1, x_p]$ from $\{x_i, y_t(x_i)\}, i = 1, 2, \dots, p$.
- (2) Decompose the fitted curves via a basis function expansion using the following model:

$$f_t(x) = \mu(x) + \sum_{k=1}^K \beta_{t,k} \phi_k(x) + e_t(x), \quad (14)$$

where $\mu(x)$ is a measure of location of $f_t(x)$, $\{\phi_k(x)\}$ is a set of orthonormal basis functions and $e_t(x) \sim N(0, v(x))$.

- (3) Fit univariate time series models to each of the coefficients $\{\beta_{t,k}\}, k = 1, \dots, K$.
- (4) Forecast the coefficients $\{\beta_{t,k}\}, k = 1, \dots, K$, for $t = n + 1, \dots, n + h$ using the fitted time series models.
- (5) Use the forecast coefficients with (3) to obtain forecasts of $f_t(x), t = n + 1, \dots, n + h$. From (1), forecasts of $f_t(x)$ are also forecasts of $y_t(x)$.
- (6) The estimated variances of the error terms in (2) and (1) are used to compute prediction intervals for the forecasts.

One important point is obviously to find the right number of basis functions, i.e. find the order K of the model. To find the order K of the model, the integrated squared forecast error ($ISFE$) is minimized on a rolling hold out sample, whereas

$$ISFE_n(h) = \int_x e_{n,h}^2(x) dx. \quad (15)$$

That is, we fit the model to data up to time t and predict the next m periods to obtain $ISFE_t(h), h = 1, \dots, m$. Then we choose K to minimize $\sum_{t=N}^{n-h} \sum_{h=1}^m ISFE_t(h)$ where N is the minimum number of observations used to fit the model.

5 The EIB - Bond, would it have been a good choice?

5.1 Fitting the model

The EIB bond was designed to hedge a annuity book of 65 year old british and welsh males in 2003. For the further analysis the data from 65 year old Austrian males, provided by the Mortality Human Database, was taken. The data are printed in Figure 17. To value the bond the point of view is 2003, even tough actual data until 2008 exists. As explained in the stylized facts part, the variance increases with ages above 95 years, hence fitting the full range would add too much noise. But through the nonparametric smoothing with weighted penalized regression splines with a monotonicity constraint, based on Wood (1994), the variance decreases. For the year 2008 were most datapoints exists the variance decreases for instance from 66% to 2.2%. The smoothed function is shown in Figure 18.

To find the right order, i.e. the right number of basis functions $\phi_k(x)$, the fitted curves was decomposed via a basis function expansion using (3). 10 different models was fitted, i.e. with 10 different orders, $K = 1, \dots, 10$, to the periods from 1947 up to 1983. Next a forecast with each model for 25 years, means up to 2008 where we have actual data was done. After that, the values of the forecasts was compared to the actual data what gave the $ISFE$ s as follows:

(1) 305.0344, (2) 305.0344, (3) 305.0773, (4) 293.5922, (5) 292.9019, (6) 285.9849, (7) 285.9849, (8) 285.9849, (9) 285.9849, (10) 285.9849, leading to a model with order 6 as the 6th $ISFE$ is lowest.

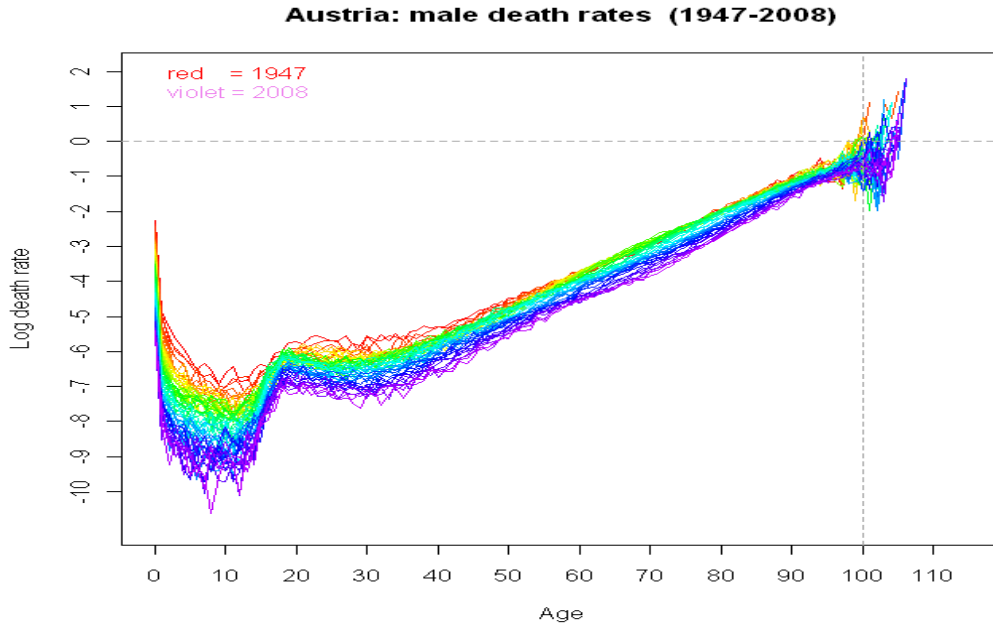


Figure 17: Logarithmic mortality rates of Austrian males 1947(upper) - 2009(lower), own calculation. Data:MHD

Figure 19 shows the main effect and the fitted $\hat{\phi}_k(x)$ in the first row and the fitted coefficients $\hat{\beta}_{t,k}$ in the second row. The model is fitted from 1947 up to 2002 for ages 0 up to 110 years.

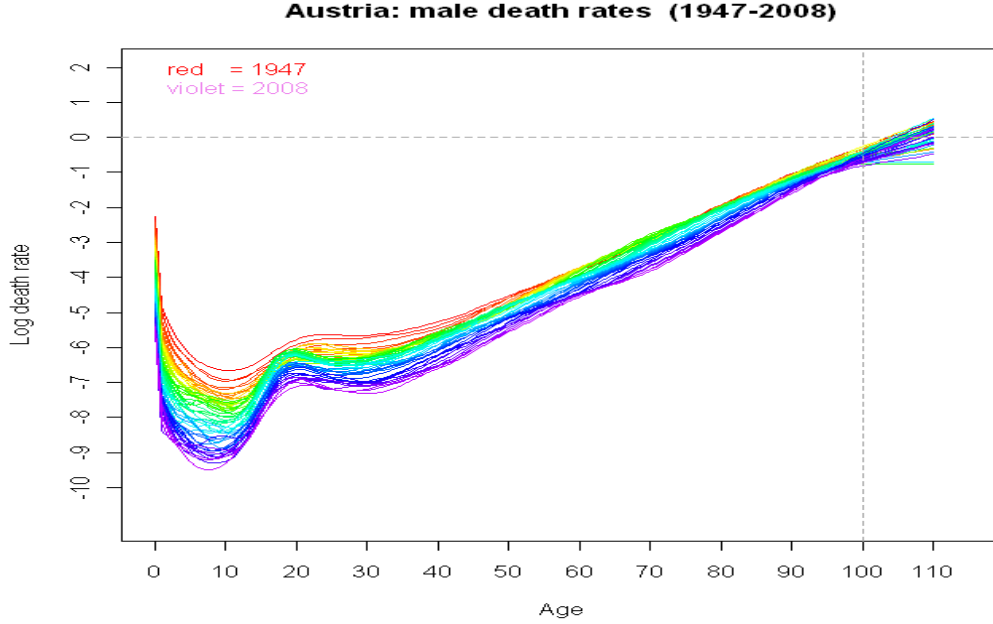


Figure 18: Smoothed mortality rates of Austrian males, own calculation.
Data:MHD

To find robust functional principal components the two-step algorithm for functional principal components as proposed by Hyndman and Ullah (2006) was used. However the six basis functions explains in sum 98.2033% of the variance in the data. The first 89.20%, the second 3.72%, the third 2.91%, the fourth 1.39%, the fifth 0.54% and the sixth explains barely 0.42%. Figure 19 shows top left the mean of the data, the other twelve pictures are the basis functions (top) with the corresponding coefficients (down). Out of Figure 19 it seems apparent that the basis functions are modelling different movements in mortality rates: $\hat{\phi}_1(x)$ models primarily the mortality changes for ages up to 20 years but most in childrens age, $\hat{\phi}_2(x)$ models primarily the changes of the very old, $\hat{\phi}_{3,5,6}(x)$ models primarily the changes in between the young and the old ages, $\hat{\phi}_4(x)$ models the difference between the young and the old. The mortality in younger years, i.e. up to 20 years, decreased over the whole period more than the mortality for the ages above. For the very old it is volatile over the whole period, the ages between the young and the very

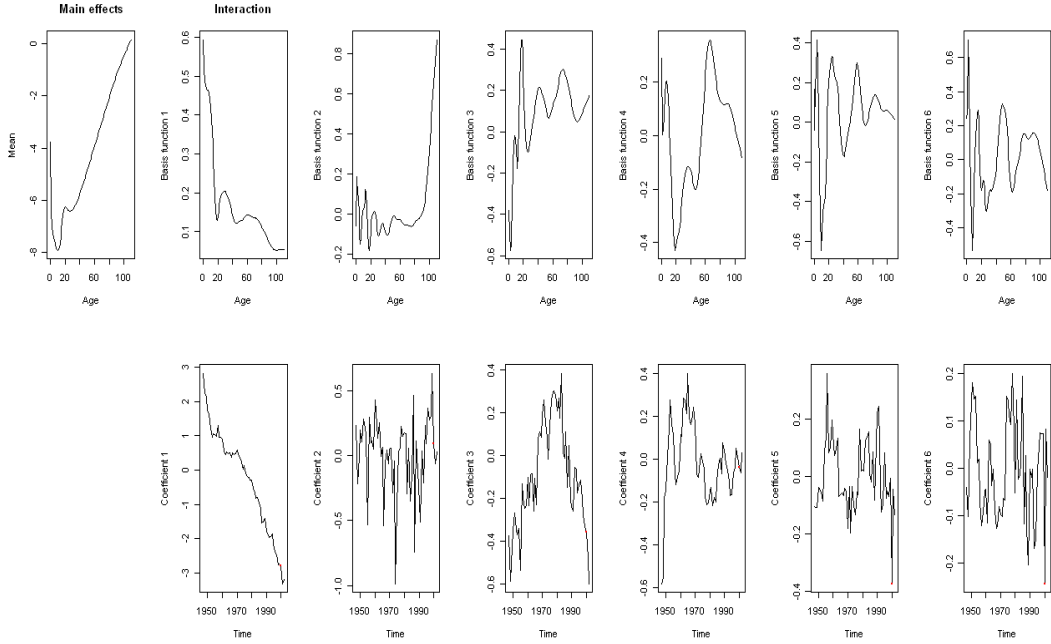


Figure 19: Basis function and coefficients of the fitted model. Austrian males, 1947 - 2002

old shows since 1980 a steady decrease in mortality whereas the difference between young and olds decreased since the 1960's.

5.2 The simulation

Next the coefficients from the fitted object are forecast using an exponential smoothing method. The forecast coefficients are then multiplied by the basis functions to obtain a forecast demographic rate curve. Based on this a Monte Carlo Simulation with 5.000 iterations out of the fitted model was done, Figure 20 shows the sample path with the mean plotted in red and the actual values from 2003 up to 2008 nearby the mean as black dots (same in all following simulations). Hence the simulation seems plausible from today's point of view as the actual data are in the fan chart.

The next step was to compute the Index $S(t)$, $t = 1, 2, \dots, 25$ out of the simulated mortality rates (q_x^s) as explained in 2.2.3.2. Through the fore-

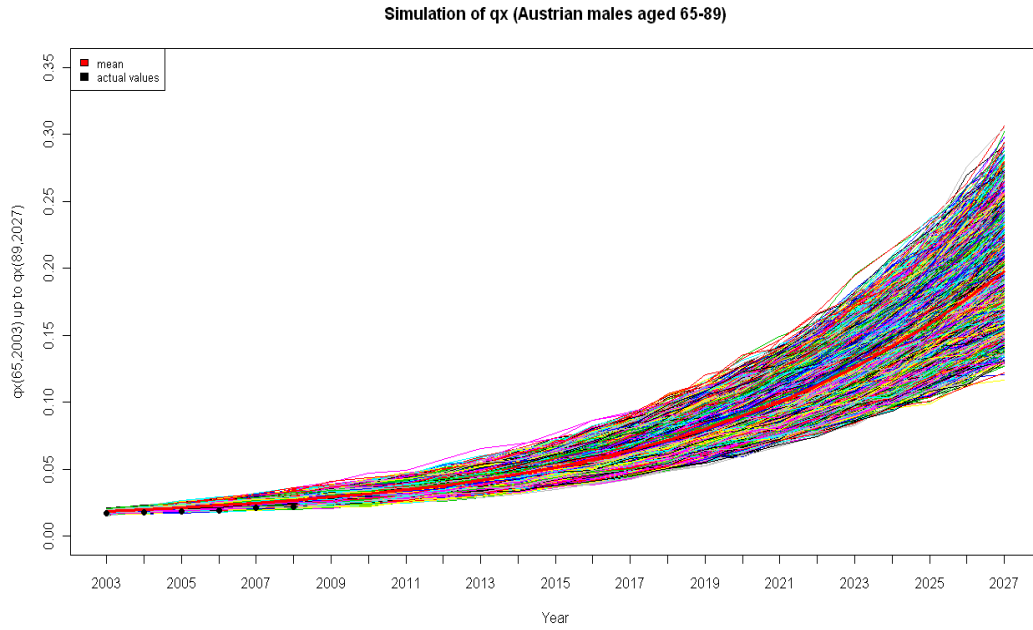


Figure 20: "*Monte Carlo Simulation*" of male mortality rates. Own calculation

going Monte Carlo Simulation 5.000 different paths of the index rates was computed. Out of the index rates the coupon rates $c(t) = S(t) \times \text{£}50m$, $t = 1, 2, \dots, 25$ was simulated as shown in Figure 21.

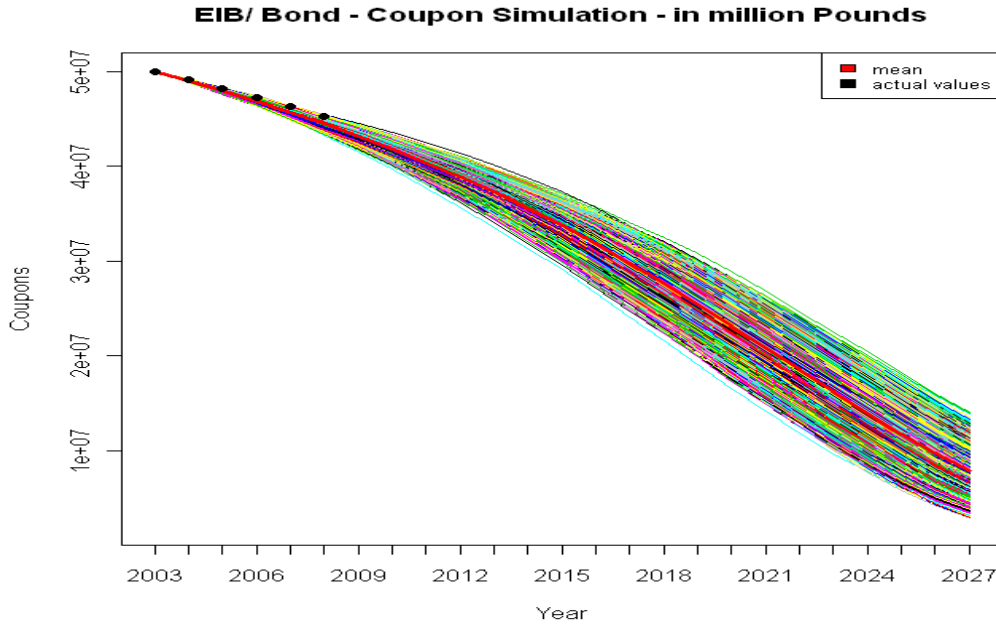


Figure 21: EIB-bond coupon payments, own calculation.

To decide whether this bond gives positive or negative returns if it is seen as a pure investment, the Net Present Value (NPV) for every single simulation was computed. As discount rate the *"secondary market rate for all emittents"* for the year 2002, provided by the Austrian Nationalbank (4.44%) was chosen. Out of this simulation 1.136 negative NPV out of 5.000 simulations was the result. The density, including a 95% confidence interval with range $[-13.707.629; 31.158.720]$ and the mean with 8.725.545 is shown in Figure 22.

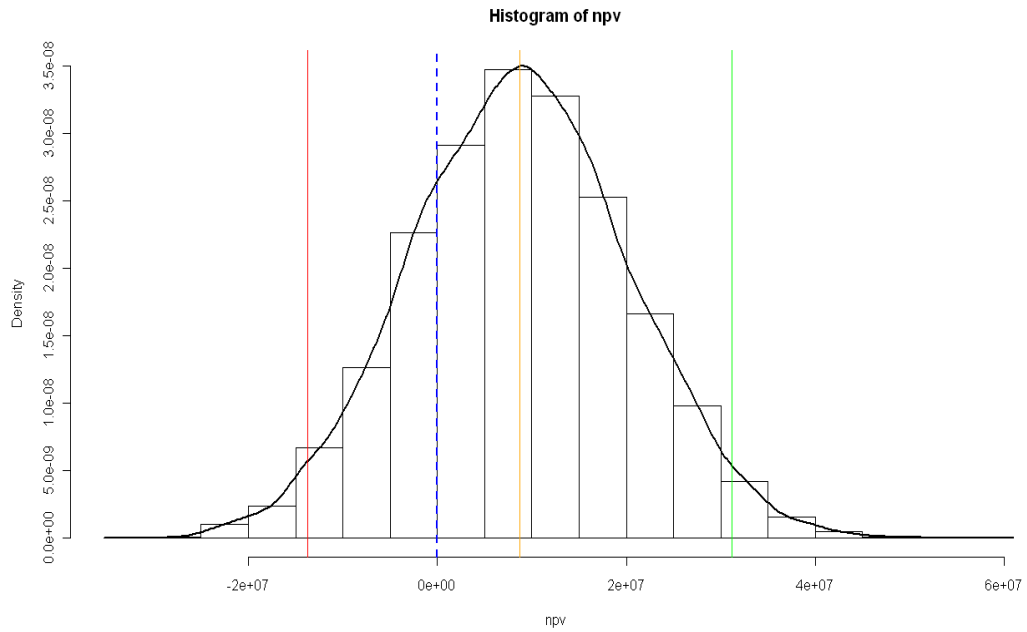


Figure 22: Density of the NPVs including 95% confidence intervall and mean, own calculation.

The QQ-plot of the NPVs, as shown in Figure 23, suggests that the NPVs are similar to a normal distribution in the middle, the left tail shows about 27 NPVs, around 0.5%, which are not following the normal distribution. This values should not be neglected.

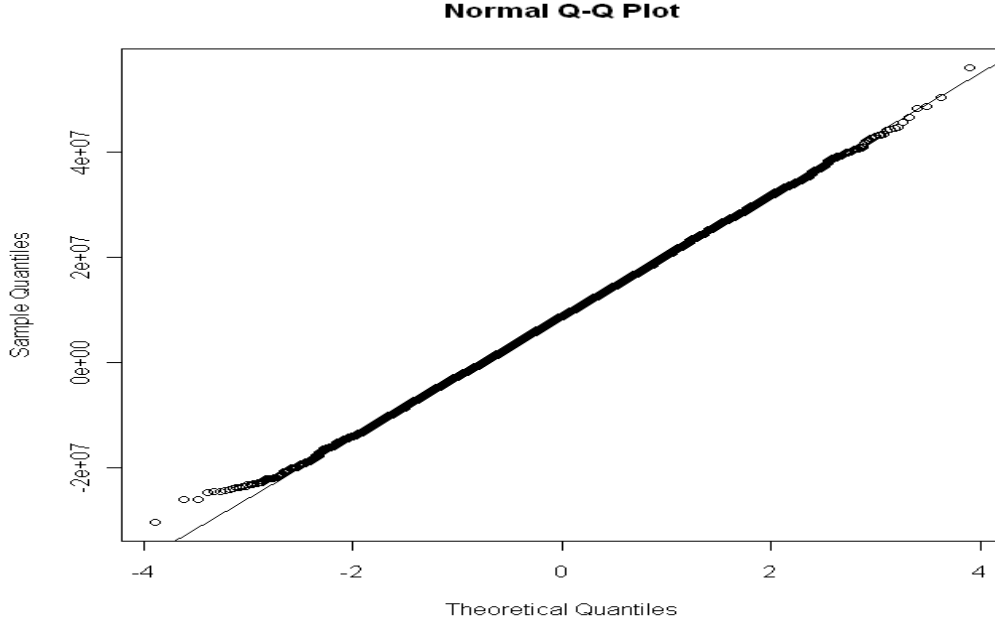


Figure 23: QQ-Plot of the NPVs, own calculation

As a second criteria the Internal Rate of Return (IRR) was chosen. The IRR states how much percent one unit of invested money grows per period on average, thus it is independent on the interest rate assumption unlike the NPV. The IRR was computed for every coupon stream. The density curve of the rates including the 95% confidence interval in the range of $[0.03701628; 0.04768879]$, the mean (orange) 0.04235254 and a line (blue) for the chosen interest rate of 0.0444, to discount the coupons for the NPV, was plotted in Figure 24. The IRR suggests that the EIB-bond is a good investment since inflation is below.

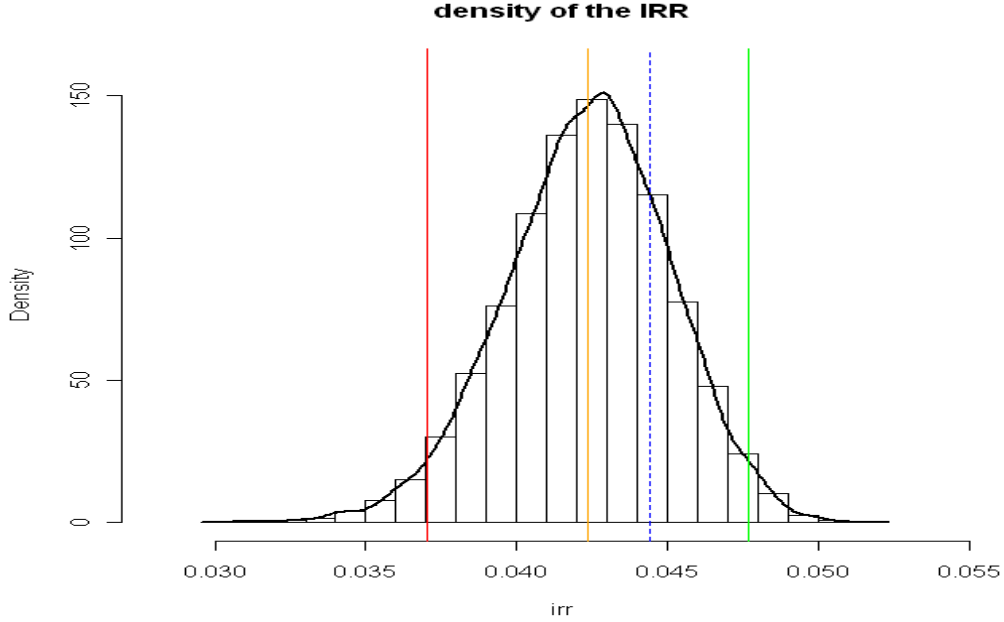


Figure 24: Density of the Internal Rate of Return from the EIB-bond. Own calculation.

5.2.1 The Valuation

To find out, whether the EIB-bond is useful as hedging instrument or not, the calculus of Net Present Value (NPV) was chosen. Actually a annuity contract is nothing more than a investment from the insurance companies point of view thus this method seems plausible as incoming (i.e., the contribution) and outgoing (i.e., the benefits) payments are given. The difference from the EIB bond to others is that the coupons are linked to a mortality index. Usually the coupon of a bond is the amount of interest paid per year expressed as a percentage of the face value of the bond. It is the interest rate that a bond issuer will pay to a bondholder, see Sullivan and Sheffrin (2003). This means the bondholder knows *ex ante* the interest rate, i.e. the value of the coupon. To overcome this difference, the coupon streams of the EIB bond was modeled and sampled as explained above. The sampling of the coupon rates should help to get rid of this lack of information concerning

the coupons. For the further analysis the simulated coupon streams $c(t)$ was scaled. This means that all simulated $c(t), t = 1, \dots, 25$ was divided by the principal payment (540m) of the EIB bond. This method provides thus the coupon stream $c(t)$ if one purchases one unit of the EIB bond.

Further should be mentioned that the only observed kind of cost in this analysis is the principal payment of the bond, all others are neglected. The reason therefore is that costs of administration, distribution and marketing are to widespread among insurance companies. The capital required (CR) is the amount of money a insurance needs as contribution to follow its liability, i.e. the annuity payment. Based on the AVOE2005R annuity table and following the equivalence principle in (1), the $CR = 12,61$. Thus the annuity provider is able to buy 12 units, as only integer values are allowed, this is called Initial Payment (IP), $IP = 0, \dots, 12$. The following analysis comprises the hedge with 0 up to 12 units of the EIB-bond. The analysis is first done without any probabilities included, in the second step a conditional probability is introduced. Usually NPV calculations are done without probabilities like in capital budgeting, but in insurance business probabilities, i.e. mortality or survival rates, are included in NPV calculation. Thus leading to the so called "Expected Net Present Value" ($E(NPV)$), see Gerber (1986).

5.2.2 Deriving the Net Present Values

The discount factor is given by

$$v^k = \frac{1}{(1+i)^k} \quad (16)$$

the discounted yearly annuity is given by

$$Z^b = \sum_{k=0}^{45} v^k \cdot 1 \quad (17)$$

including the contribution (CR) and the (IP) in 2002, i.e. period 0, gives the NPV

$$NPV^{bond} = (CR - IP) - \sum_{k=0}^{45} v^k \cdot 1 + \sum_{k=0}^{25} v^k \cdot c(k). \quad (18)$$

One should pay attention to the limited bond-duration of 25 years. As criteria to check the hedging value of the bond the scenario $IP = 0$ is compared to the scenarios $IP = 1, \dots, 12$ in Figure 26 and 27. The scenario $IP = 0$ is given by the orange line, it depends in this part of the analysis not on any probability, it should be interpreted as reference line. The topleft graphic is the hedge through one unit bond increasing up to 12 units in bottomright. Both graphs suggests that only the hedge with one unit of bond extends the duration with 100%. In Figure 26 the NPVs are plotted with respect to time, the broad bands show the values out of the simulation as the bond depends on mortality rates (5.000 NPV for each scenario).

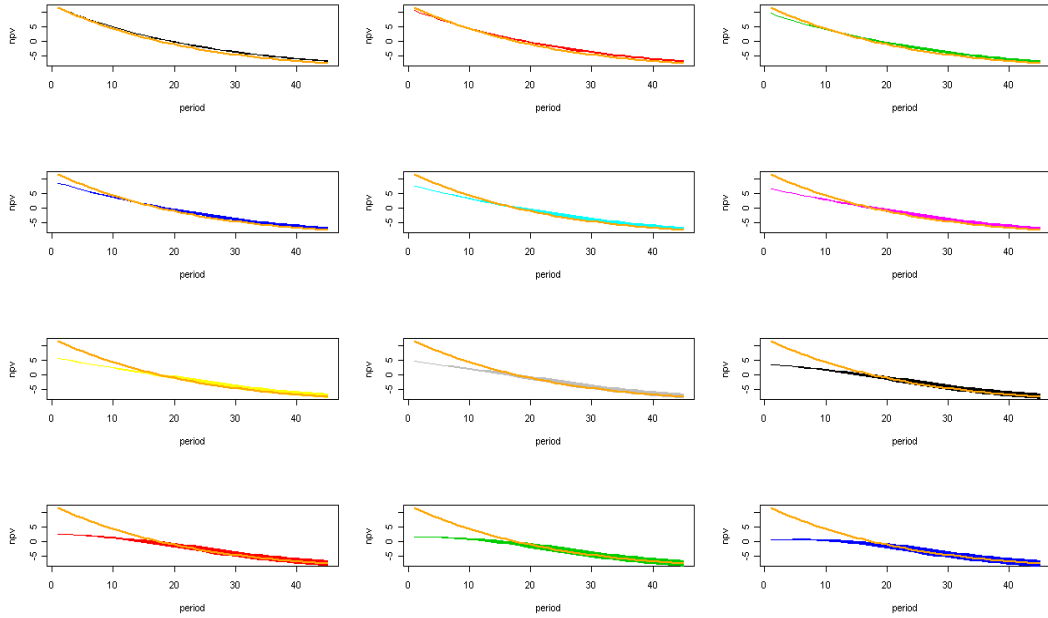


Figure 25: NPV of the cashflow stream, own calculation

In Figure 27 the densities of the NPV s are plotted. The colored broad bands shows the 5.000 possible NPV^{bond} densities out of the simulation for each scenario, i.e. $IP = 1, \dots, 12$, compared to the density of the NPV , orange line.

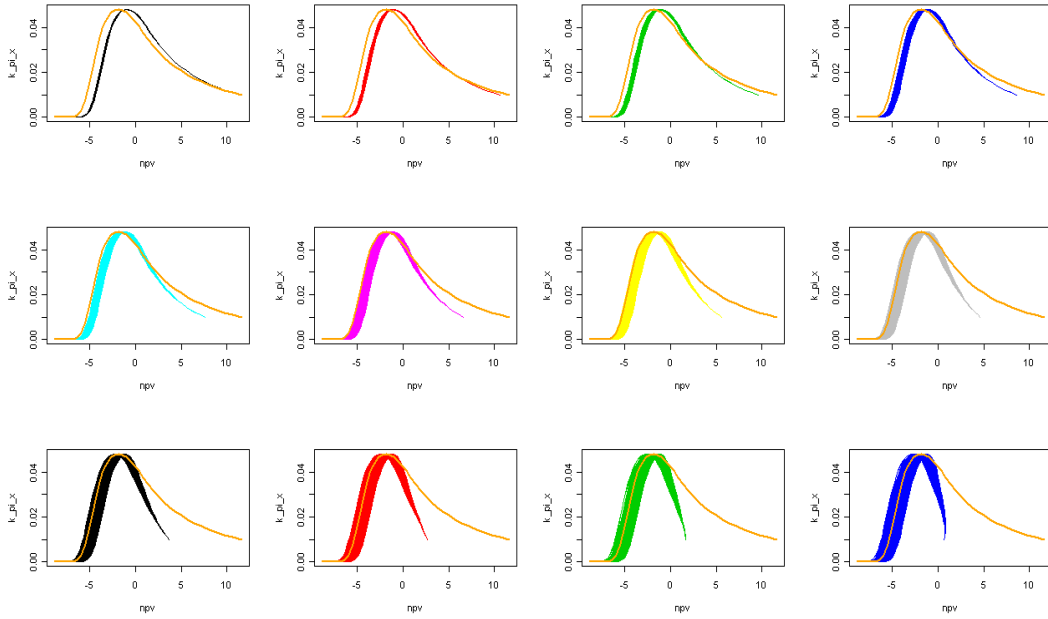


Figure 26: Densities of the cashflows, own calculation

The extension of the NPV is superior in the first picture. But to decide whether the bond is a good opportunity or not it is necessary to introduce the $E(NPV)$.

5.2.3 Deriving the Expected Net Present Values, with the underlying conditional probability

The usual q_x gives the probability of a person aged x to die before $x + 1$. But to value this bond, several survival scenarios of the underlying annuitant are necessary. This aim could be reached by the introduction of a conditional mortality probability $({}_k p_x^h)$, which is based on the simulated mortality rates (q_x^s) , the superscript $(.^s)$ indicates for all rates that they are out of the foregone simulation. To calculate this probability the unconditional probabilities need to be introduced:

1. $p_x^s = 1 - q_x^s$ is the probability to survive until $x + 1$.
2. ${}_k p_x^s = p_x^s \cdot p_{x+1}^s \cdots p_{x+k-1}^s$ is the k year survival probability.

The indices are in the range: $x = 65, k = 0, \dots, 45$ and $h = 0, \dots, 45$. Finally the conditional probability is given through

$${}_k p_x^h = \frac{{}_k p_x^s}{{}_h p_x^s} \text{ iff } k > h \text{ and } {}_k p_x^h = 1 \text{ iff } k \leq h. \quad (19)$$

It is the probability that a life aged x survives up to $x + k$ given the surviving at least to $x + h$. The index h is thus the value which indicates longevity, for example if $h = 30$ and $k = 30$ the person reached the age of 95 for sure, i.e. ${}_{30}p_{65}^{30} = 1$.

Including ${}_k p_x^h$ in (18) gives thus the $E_h(NPV)$ for each scenario $h = 0, \dots, 45$.

$$E_h(NPV) = (CR - IP) - \sum_{k=0}^{45} v^k \cdot 1 \cdot {}_k p_x^h + \sum_{k=0}^{25} v^k \cdot c_k, \quad h = 0, \dots, 45. \quad (20)$$

For the interpretation it must be mentioned that this probability in combination with the different buying scenarios leads to a explosion of the data.

This means in fact that for every scenario $h = 0, \dots, 45$, a subscenario, determined by the value of $IP = 0, \dots, 12$ is simulated. To keep it clear, the result for every h is a matrix with 5000 rows (depending on the 5000 ${}_k p_x^h$) and 13 columns (depending on the 13 IP s). To value the usefulness of the EIB bond, it is necessary to define longevity. In the case of a 65 year old Austrian male, it should be defined in that way: A life aged 65 surviving the expected residual lifetime $e(65)$ out of the 2003 mortality table from the Statistik Austria. In our case $e(65)$ is about 17 years, thus, if $h > 17$ we speak from longevity.

The analysis gives surprising results. The means of the $E_h(NPV)$, $IP > 0$ are all better of than the means of $E_h(NPV)$, $IP = 0$. This suggests that the purchase of the bond is in every scenario superior. The value of the means itself increases with increasing IP for all scenarios h . Using the means of $E_h(NPV)$ as decision criteria, the purchase of twelve units of the bond, i.e. $IP = 12$, would be superior, see graphic 27.

The $E_h(NPV)$, $IP = 0$ turns negative for $h = 6$. The negative $E_h(NPV)$ could be extended for one period if and only if $IP = 12$, it should be noted that for $h = 7$, $E_7(NPV) = -0.01$ is rather close to zero. At $h = 17$, the predefined border to longevity, the $E_{17}(NPV)$, $IP = 0$ is 16.25% worse off than the $E_{17}(NPV)$, $IP = 12$; if the Austrian man reaches the age of 100, i.e. $h = 35$, the $E_{35}(NPV)$, $IP = 0$ is 5.86% worse off than the $E_{35}(NPV)$, $IP = 12$.

The boxplots in Figure 28-35 shows the distribution of the $E_h(NPV)$ for $h = 0, \dots, 45$ and $IP = 0, \dots, 12$. The boxplots confirm the foregone analysis. The median, i.e. the central line in the box, is in all scenarios better off, if one buys at least one unit of the bond, i.e. $IP > 0$. The superior rank of $IP = 12$ is also confirmed by the boxplots. If $IP = 12$, even the boxes, i.e. the lower and upper quartile of the distribution, are better of than the median of $E_h(NPV)$, $IP = 0$.

It should be mentioned that the $E_h(NPV)$ would be probably more better off for a real annuity stock. The adverse selection suggests that the q_x out of a real annuity stock are lower than the whole population mortality rates used in this analysis. One should keep in mind that only a healthy and cautious

human being would sign a annuity contract.

This all suggests that the bond would have been a good choice, at least for 65 year old Austrian males.

	IP=0	IP=1	IP=2	IP=3	IP=4	IP=5	IP=6	IP=7	IP=8	IP=9	IP=10	IP=11	IP=12
h=0	1,03	1,05	1,08	1,11	1,14	1,16	1,19	1,22	1,25	1,27	1,30	1,33	1,36
h=1	0,83	0,86	0,89	0,92	0,94	0,97	1,00	1,03	1,05	1,08	1,11	1,14	1,16
h=2	0,64	0,67	0,69	0,72	0,75	0,78	0,80	0,83	0,86	0,89	0,91	0,94	0,97
h=3	0,44	0,47	0,50	0,53	0,55	0,58	0,61	0,64	0,66	0,69	0,72	0,75	0,77
h=4	0,25	0,28	0,30	0,33	0,36	0,39	0,41	0,44	0,47	0,49	0,52	0,55	0,58
h=5	0,05	0,08	0,11	0,13	0,16	0,19	0,22	0,24	0,27	0,30	0,33	0,35	0,38
h=6	-0,14	-0,12	-0,09	-0,06	-0,03	-0,01	0,02	0,05	0,08	0,10	0,13	0,16	0,19
h=7	-0,34	-0,31	-0,28	-0,26	-0,23	-0,20	-0,17	-0,15	-0,12	-0,09	-0,06	-0,04	-0,01
h=8	-0,53	-0,51	-0,48	-0,45	-0,42	-0,40	-0,37	-0,34	-0,31	-0,29	-0,26	-0,23	-0,21
h=9	-0,73	-0,70	-0,67	-0,65	-0,62	-0,59	-0,57	-0,54	-0,51	-0,48	-0,46	-0,43	-0,40
h=10	-0,93	-0,90	-0,87	-0,84	-0,82	-0,79	-0,76	-0,73	-0,71	-0,68	-0,65	-0,62	-0,60
h=11	-1,12	-1,10	-1,07	-1,04	-1,01	-0,99	-0,96	-0,93	-0,90	-0,88	-0,85	-0,82	-0,80
h=12	-1,32	-1,30	-1,27	-1,24	-1,21	-1,19	-1,16	-1,13	-1,10	-1,08	-1,05	-1,02	-1,00
h=13	-1,53	-1,50	-1,47	-1,44	-1,42	-1,39	-1,36	-1,33	-1,31	-1,28	-1,25	-1,22	-1,20
h=14	-1,73	-1,70	-1,68	-1,65	-1,62	-1,59	-1,57	-1,54	-1,51	-1,48	-1,46	-1,43	-1,40
h=15	-1,94	-1,91	-1,88	-1,86	-1,83	-1,80	-1,77	-1,75	-1,72	-1,69	-1,66	-1,64	-1,61
h=16	-2,15	-2,12	-2,09	-2,07	-2,04	-2,01	-1,98	-1,96	-1,93	-1,90	-1,87	-1,85	-1,82
h=17	-2,36	-2,33	-2,31	-2,28	-2,25	-2,22	-2,20	-2,17	-2,14	-2,11	-2,09	-2,06	-2,03
h=18	-2,57	-2,55	-2,52	-2,49	-2,47	-2,44	-2,41	-2,38	-2,36	-2,33	-2,30	-2,27	-2,25
h=19	-2,79	-2,76	-2,74	-2,71	-2,68	-2,65	-2,63	-2,60	-2,57	-2,55	-2,52	-2,49	-2,46
h=20	-3,01	-2,98	-2,96	-2,93	-2,90	-2,87	-2,85	-2,82	-2,79	-2,76	-2,74	-2,71	-2,68
h=21	-3,23	-3,20	-3,18	-3,15	-3,12	-3,09	-3,07	-3,04	-3,01	-2,98	-2,96	-2,93	-2,90
h=22	-3,45	-3,42	-3,40	-3,37	-3,34	-3,31	-3,29	-3,26	-3,23	-3,20	-3,18	-3,15	-3,12
h=23	-3,67	-3,64	-3,62	-3,59	-3,56	-3,54	-3,51	-3,48	-3,45	-3,43	-3,40	-3,37	-3,34
h=24	-3,89	-3,87	-3,84	-3,81	-3,78	-3,76	-3,73	-3,70	-3,68	-3,65	-3,62	-3,59	-3,57
h=25	-4,12	-4,09	-4,06	-4,03	-4,01	-3,98	-3,95	-3,92	-3,90	-3,87	-3,84	-3,81	-3,79
h=26	-4,34	-4,31	-4,28	-4,26	-4,23	-4,20	-4,17	-4,15	-4,12	-4,09	-4,06	-4,04	-4,01
h=27	-4,56	-4,53	-4,50	-4,47	-4,45	-4,42	-4,39	-4,37	-4,34	-4,31	-4,28	-4,26	-4,23
h=28	-4,77	-4,75	-4,72	-4,69	-4,67	-4,64	-4,61	-4,58	-4,56	-4,53	-4,50	-4,47	-4,45
h=29	-4,99	-4,96	-4,94	-4,91	-4,88	-4,85	-4,83	-4,80	-4,77	-4,74	-4,72	-4,69	-4,66
h=30	-5,20	-5,17	-5,15	-5,12	-5,09	-5,06	-5,04	-5,01	-4,98	-4,96	-4,93	-4,90	-4,87
h=31	-5,41	-5,38	-5,36	-5,33	-5,30	-5,27	-5,25	-5,22	-5,19	-5,16	-5,14	-5,11	-5,08
h=32	-5,61	-5,59	-5,56	-5,53	-5,50	-5,48	-5,45	-5,42	-5,39	-5,37	-5,34	-5,31	-5,28
h=33	-5,81	-5,78	-5,76	-5,73	-5,70	-5,67	-5,65	-5,62	-5,59	-5,57	-5,54	-5,51	-5,48
h=34	-6,00	-5,98	-5,95	-5,92	-5,89	-5,87	-5,84	-5,81	-5,78	-5,76	-5,73	-5,70	-5,68
h=35	-6,19	-6,16	-6,14	-6,11	-6,08	-6,05	-6,03	-6,00	-5,97	-5,94	-5,92	-5,89	-5,86
h=36	-6,37	-6,34	-6,32	-6,29	-6,26	-6,24	-6,21	-6,18	-6,15	-6,13	-6,10	-6,07	-6,04
h=37	-6,55	-6,52	-6,49	-6,46	-6,44	-6,41	-6,38	-6,35	-6,33	-6,30	-6,27	-6,25	-6,22
h=38	-6,72	-6,69	-6,66	-6,63	-6,61	-6,58	-6,55	-6,52	-6,50	-6,47	-6,44	-6,42	-6,39
h=39	-6,88	-6,85	-6,83	-6,80	-6,77	-6,74	-6,72	-6,69	-6,66	-6,63	-6,61	-6,58	-6,55
h=40	-7,04	-7,01	-6,98	-6,96	-6,93	-6,90	-6,87	-6,85	-6,82	-6,79	-6,76	-6,74	-6,71
h=41	-7,19	-7,16	-7,14	-7,11	-7,08	-7,05	-7,03	-7,00	-6,97	-6,94	-6,92	-6,89	-6,86
h=42	-7,34	-7,31	-7,28	-7,26	-7,23	-7,20	-7,17	-7,15	-7,12	-7,09	-7,07	-7,04	-7,01
h=43	-7,48	-7,45	-7,43	-7,40	-7,37	-7,34	-7,32	-7,29	-7,26	-7,23	-7,21	-7,18	-7,15
h=44	-7,62	-7,59	-7,56	-7,53	-7,51	-7,48	-7,45	-7,42	-7,40	-7,37	-7,34	-7,31	-7,29
h=45	-7,73	-7,70	-7,67	-7,65	-7,62	-7,59	-7,56	-7,54	-7,51	-7,48	-7,45	-7,43	-7,40

Figure 27: EIB-bond: Ranking of the means from the $E_h(NPV)$, $h = 0, \dots, 45$ and $IP = 0, \dots, 12$

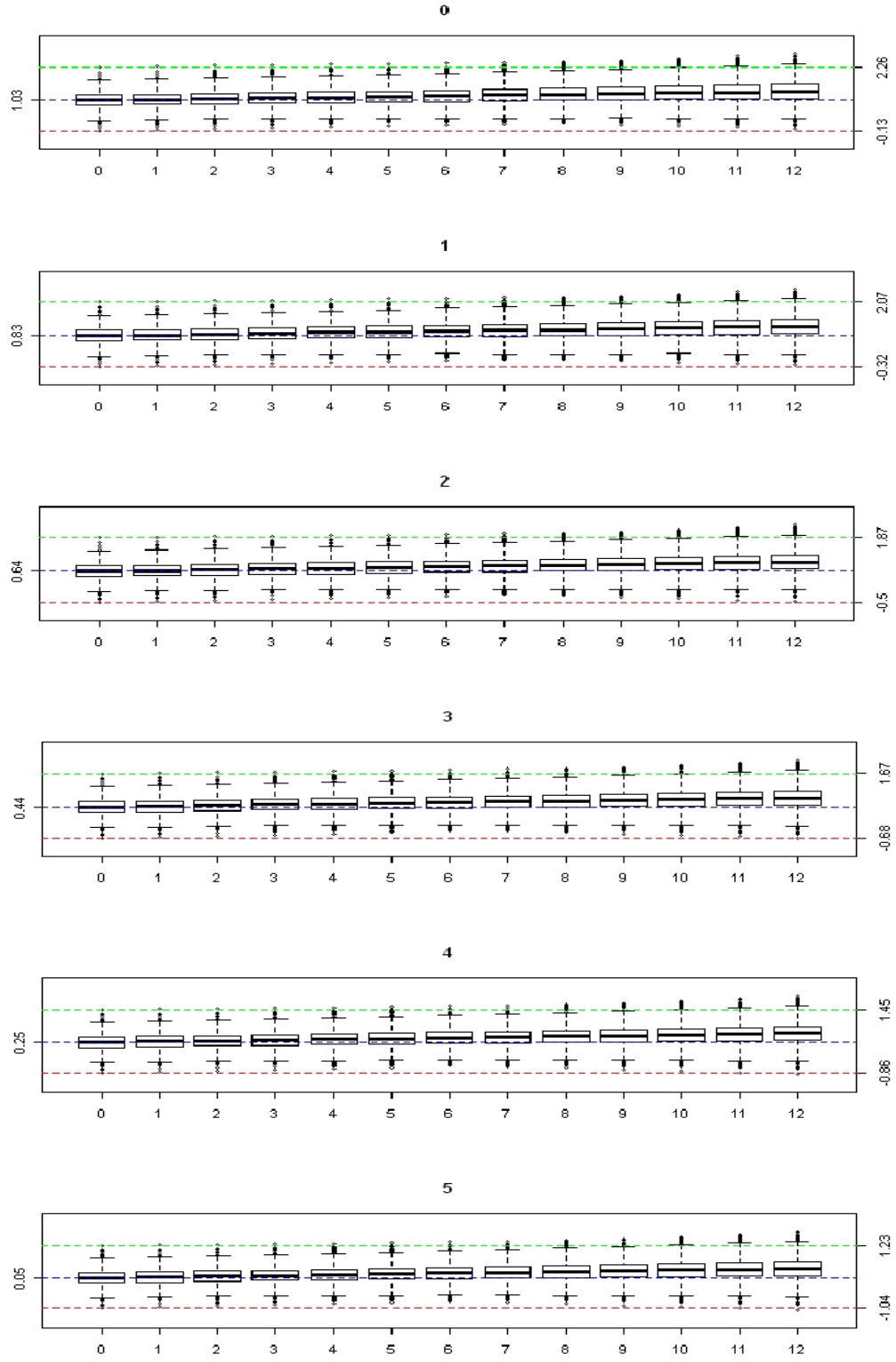


Figure 28: Boxplots of the $E_h(NPV)$, $h = 0, \dots, 5$ and $IP = 0, \dots, 12$ on the x -axis. $Mean(blue)$, $Min(red)$ and $Max(green)$ values of $IP = 0$ included.

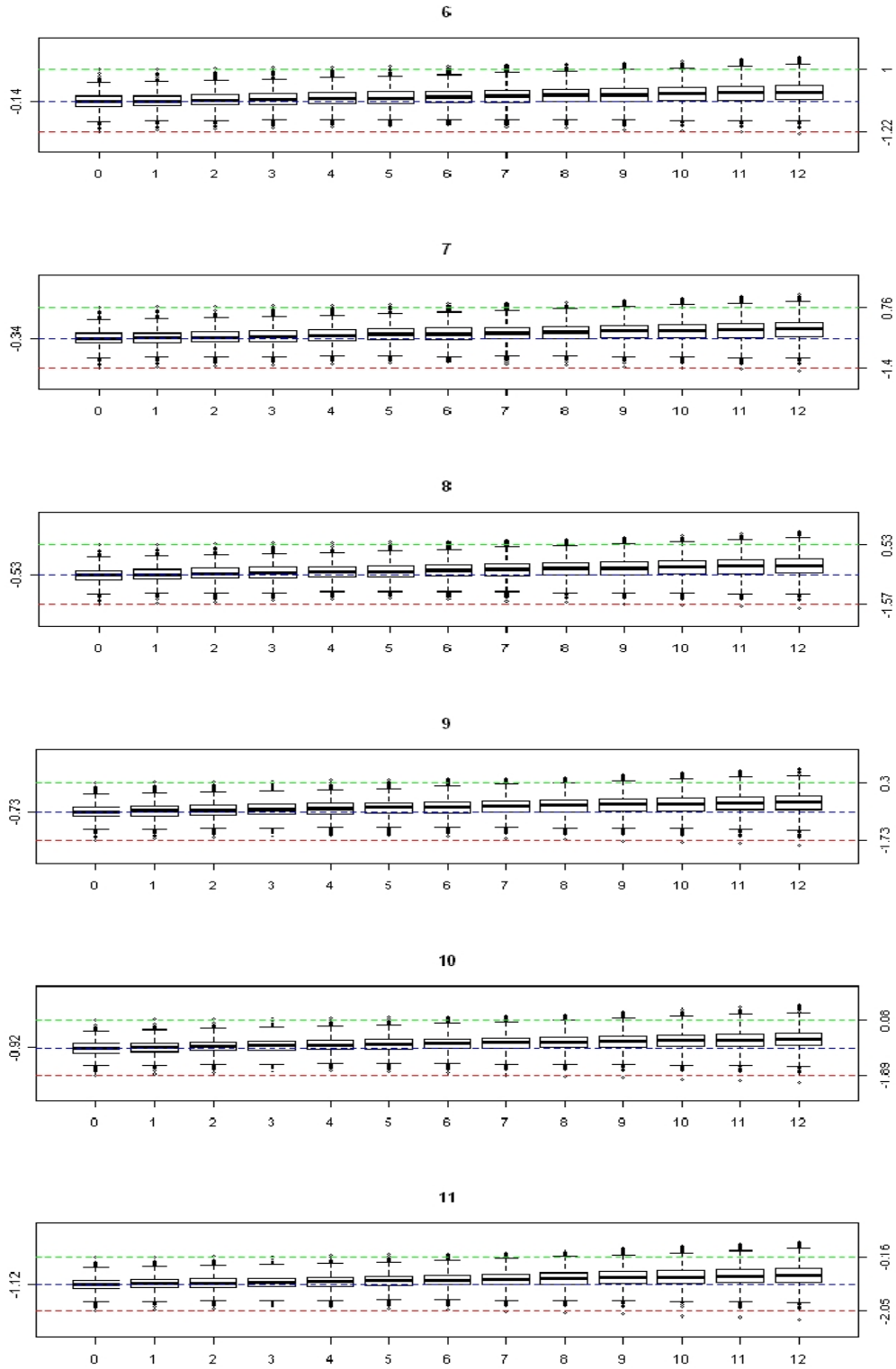


Figure 29: Boxplots of the $E_h(NPV)$, $h = 6, \dots, 11$ and $IP = 0, \dots, 12$ on the x -axis. *Mean(blue)*, *Min(red)* and *Max(green)* values of $IP = 0$ included.

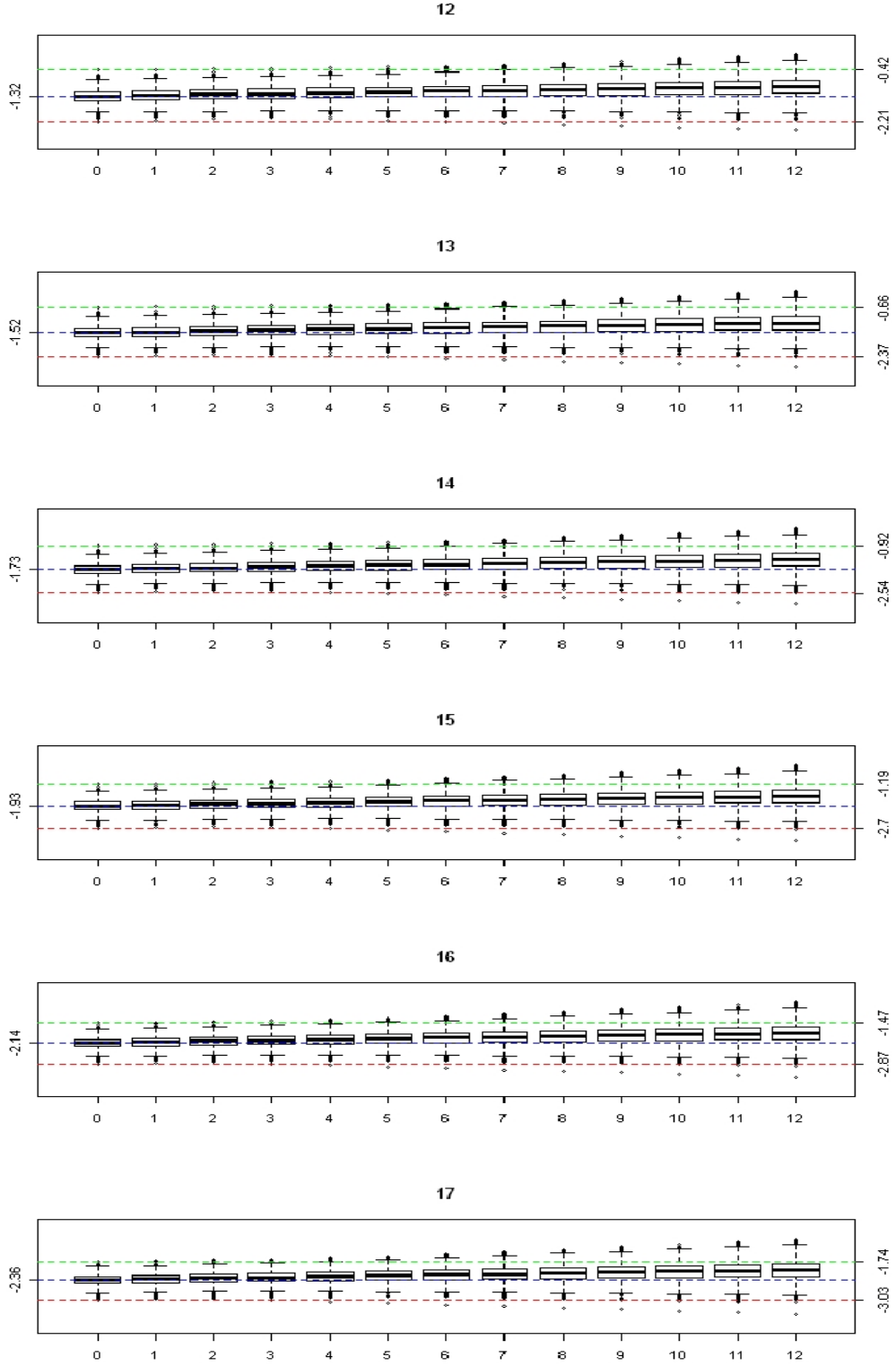


Figure 30: Boxplots of the $E_h(NPV)$, $h = 12, \dots, 17$ and $IP = 0, \dots, 12$ on the x -axis. $Mean(blue)$, $Min(red)$ and $Max(green)$ values of $IP = 0$ included.

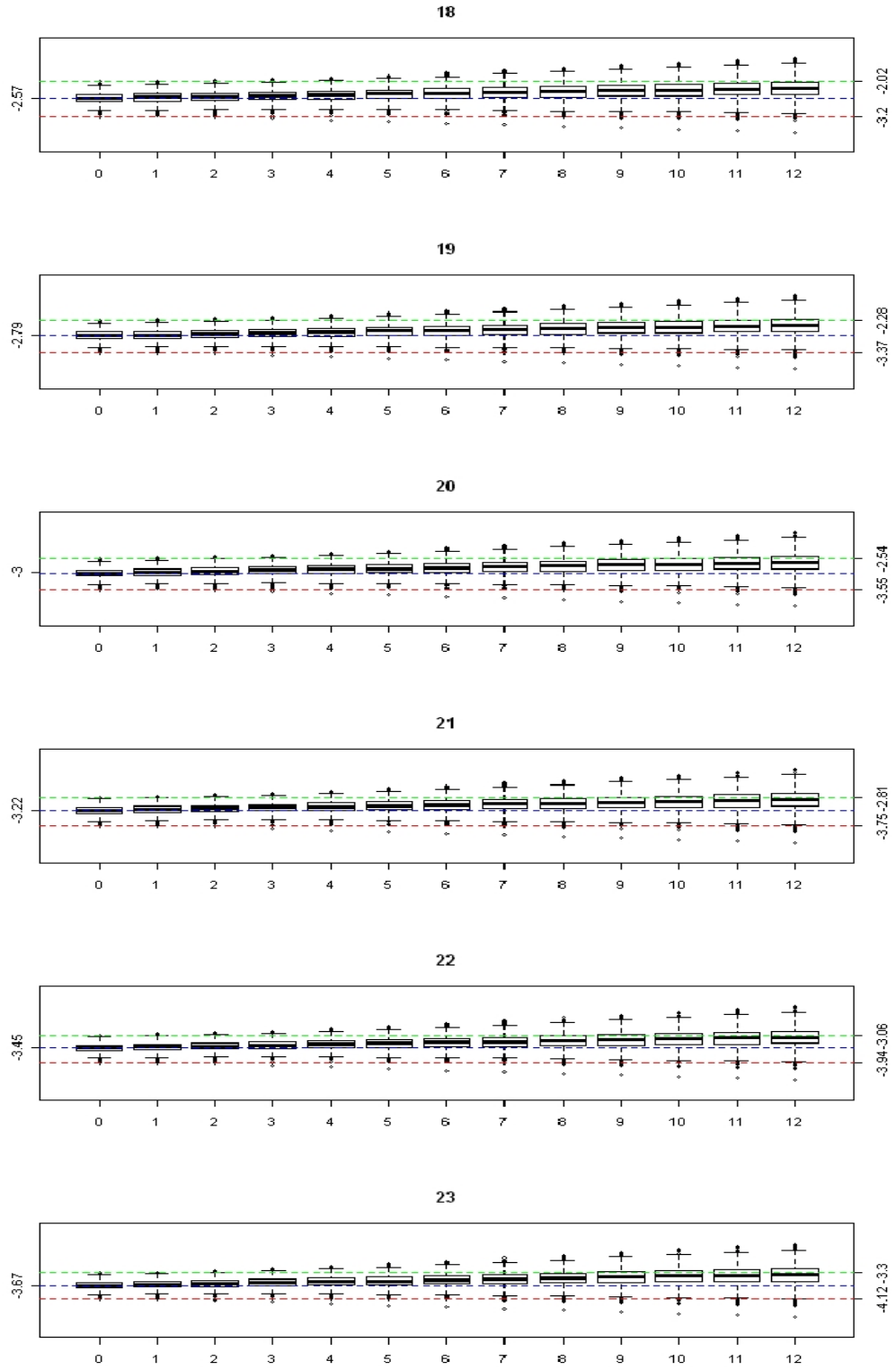


Figure 31: Boxplots of the $E_h(NPV)$, $h = 18, \dots, 23$ and $IP = 0, \dots, 12$ on the x - axis. *Mean(blue)*, *Min(red)* and *Max(green)* values of $IP = 0$ included.

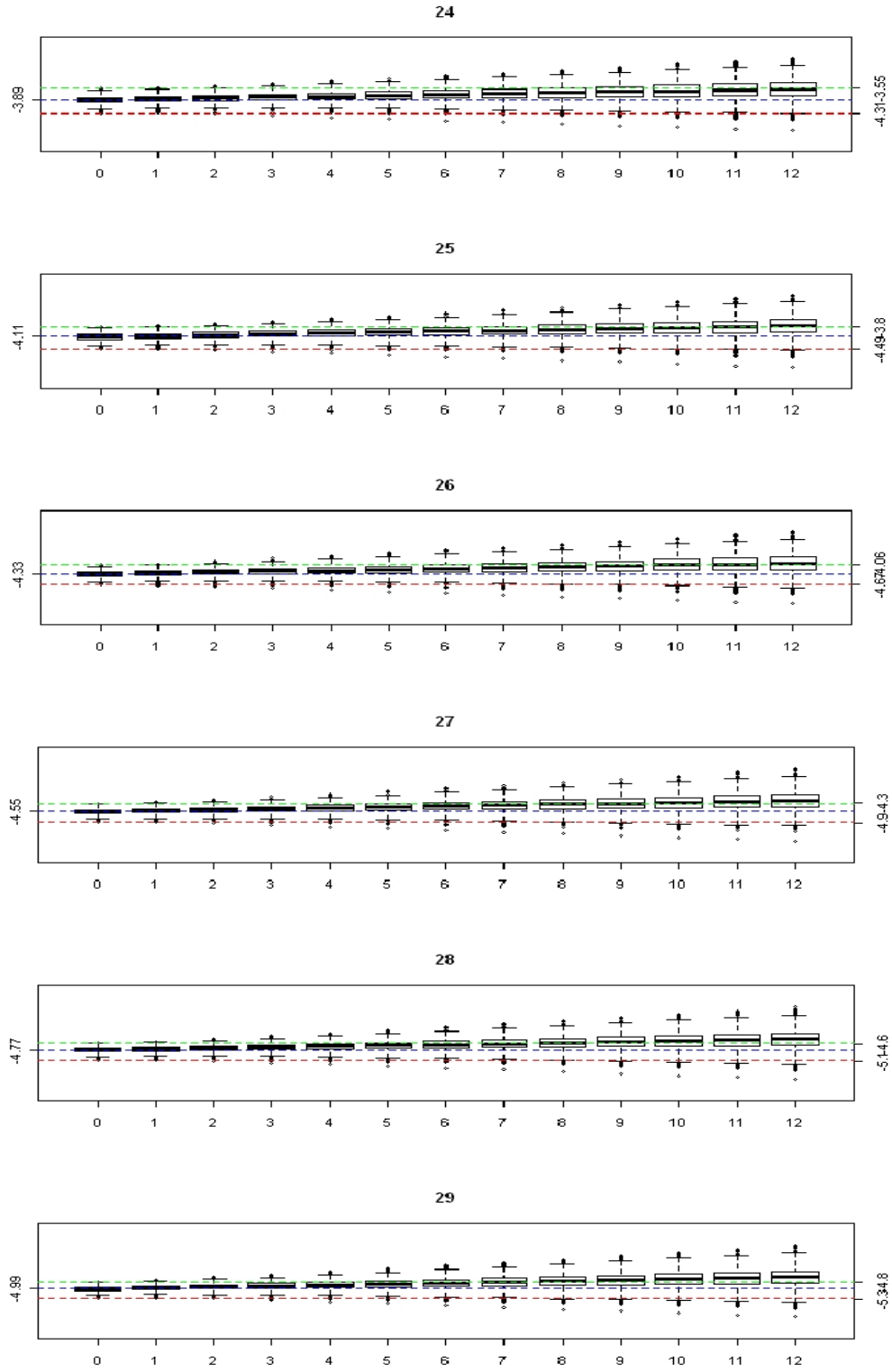


Figure 32: Boxplots of the $E_h(NPV)$, $h = 24, \dots, 29$ and $IP = 0, \dots, 12$ on the x -axis. Mean(blue), Min(red) and Max(green) values of $IP = 0$ included.

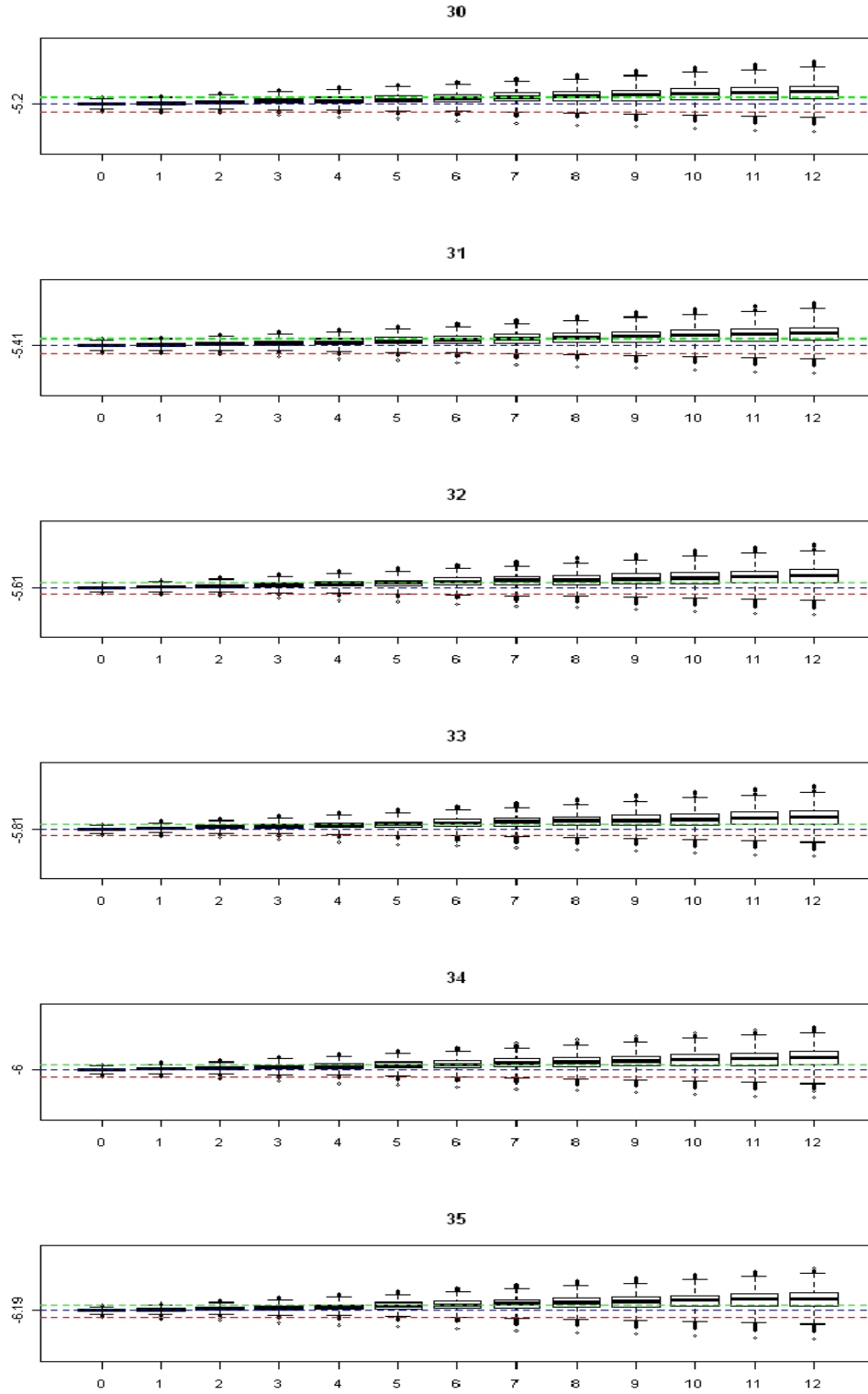


Figure 33: Boxplots of the $E_h(NPV)$, $h = 30, \dots, 35$ and $IP = 0, \dots, 12$ on the x -axis. *Mean(blue)*, *Min(red)* and *Max(green)* values of $IP = 0$ included.

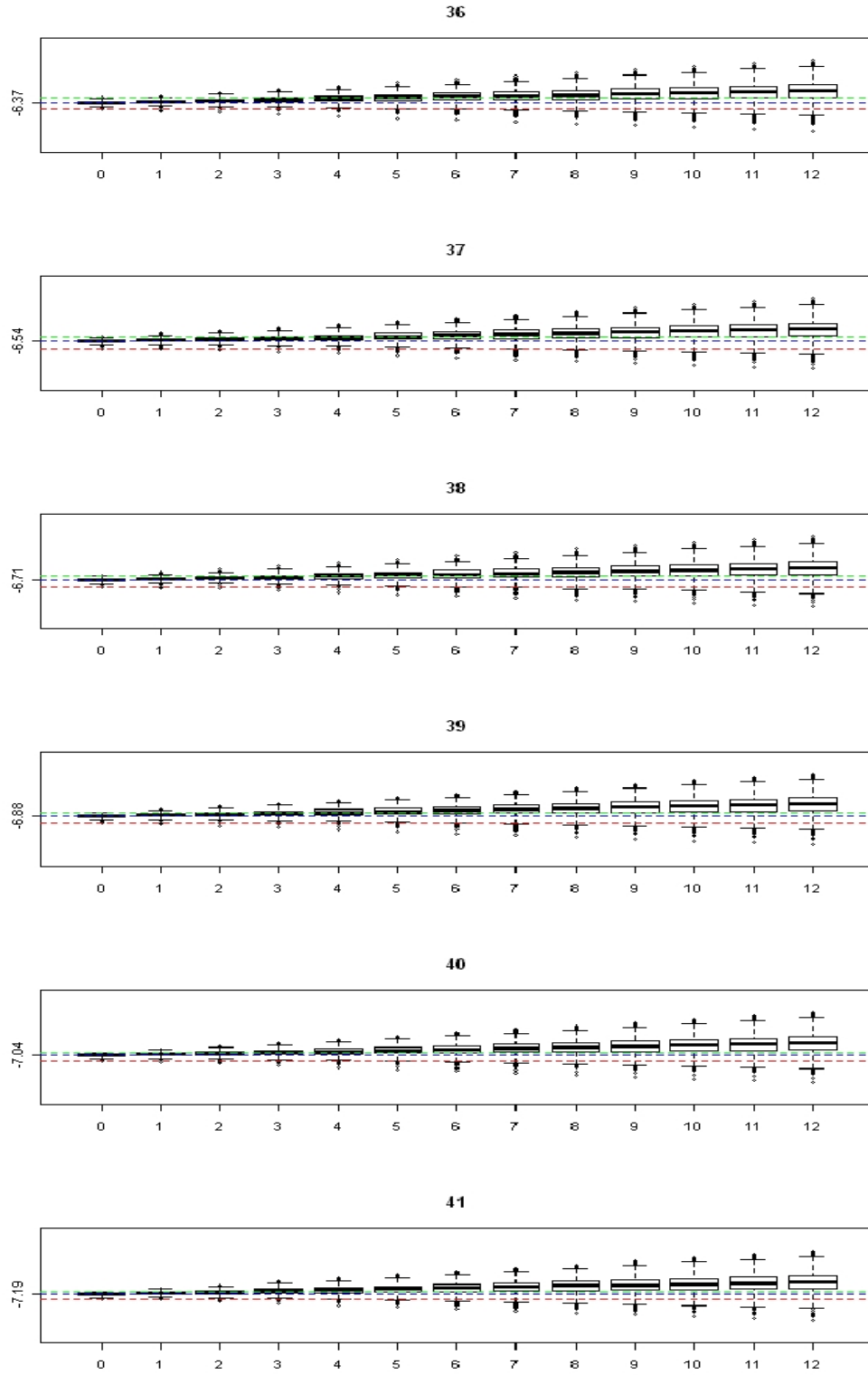


Figure 34: Boxplots of the $E_h(NPV)$, $h = 36, \dots, 41$ and $IP = 0, \dots, 12$ on the x -axis. Mean(blue), Min(red) and Max(green) values of $IP = 0$ included.

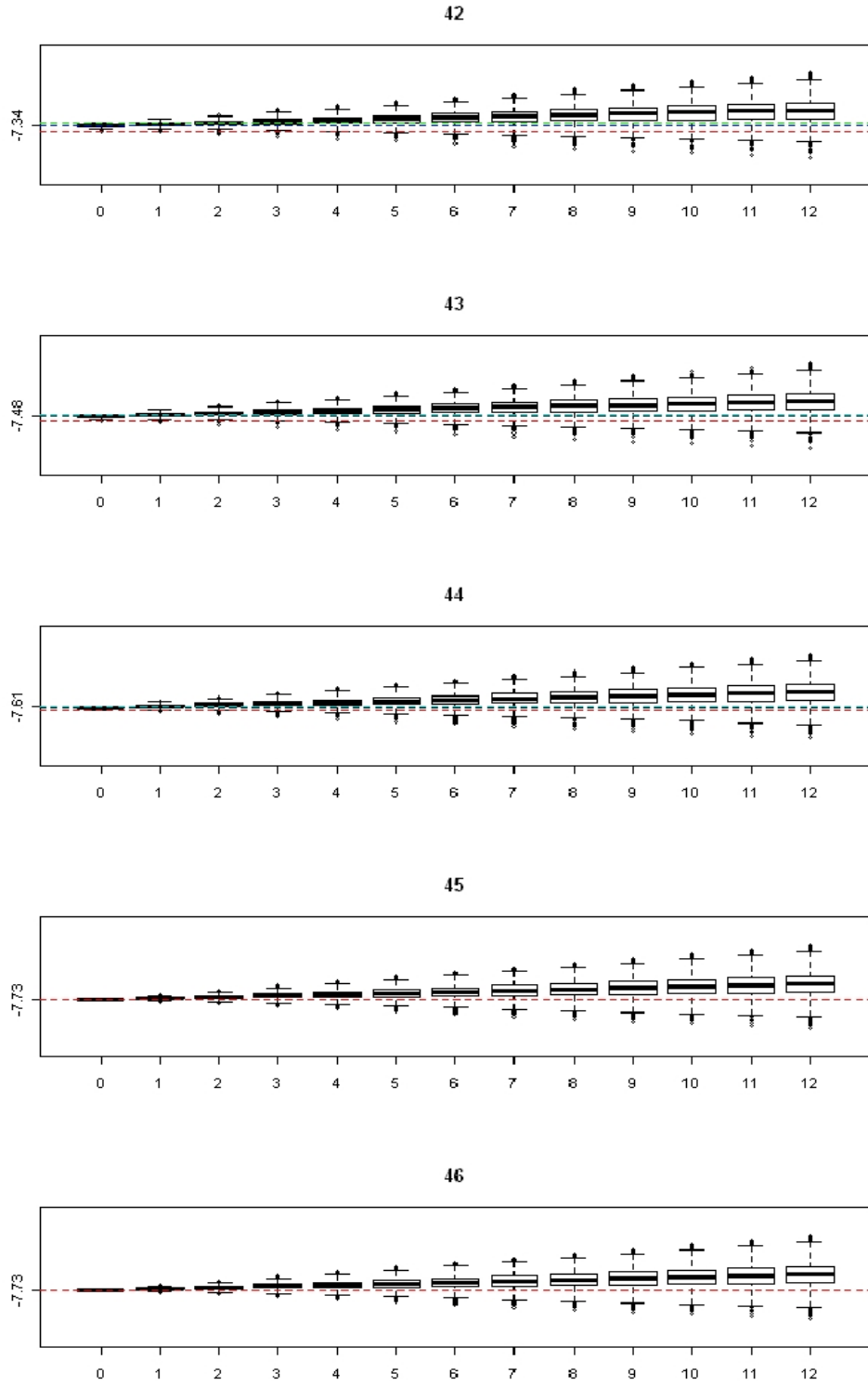


Figure 35: Boxplots of the $E_h(NPV)$, $h = 42, \dots, 46$ and $IP = 0, \dots, 12$ on the x -axis. $Mean(blue)$, $Min(red)$ and $Max(green)$ values of $IP = 0$ included.

6 Simulating a hedge with a *q-forward*

6.1 Deriving the price

In section 2.3.3 the mechanisms behind a *q-forward* contract are explained in detail. In this chapter the focus is on comparing a *q-forward* contract in contrast to the EIB-bond as hedging instrument. The mortality rates are the same, i.e. a 65 year old Austrian male. Further the simulation started from 2003 on, as it was done with the EIB bond. As first step it is necessary to derive the forward rate of such a contract, i.e. the price. Loeys et al. (2007), from JP-Morgans "Global Market Strategy", gives in his article a distinctive instruction how this is performed by JP-Morgan. As a reference point, Loeys et al. (2007) consider the historical volatility $\sigma(q_x)$ of the relative changes in mortality rates and the forecasts produced by the Lee-Carter model. Since longevity risk is virtually uncorrelated with other market risks, Loeys et al. (2007) argue that the required Sharpe ratio on *q-forwards* should be lower than the one available for riskier assets classes such as equities, but high enough to attract investors to the market. They suggest an *annualized Sharpe ratio* of 0.25, see Biffis and Blake (2009). This description was taken as given and applied to Austrian data in the following way:

- JP-Morgan requires an *annualized Sharpe-Ratio* of 0.25, whereas the *Sharpe-Ratio*¹⁹ is defined as

$$S = \frac{E(R - R_f)}{\sqrt{\text{var}(R - R_f)}} \quad (21)$$

where R is the asset return and R_f is the risk free rate of return (benchmark). In this version, the ratio indicates the expected differential return per unit of risk associated with the differential return, see Sharpe (1966 and 1975).

- The *annualized risk* is given through

$$R_a = \sigma(q_x) \cdot q_{ex}, \text{ where } q_{ex} \text{ denotes the expected future mortality.} \quad (22)$$

¹⁹also known as Reward-to-Variability-Ratio

- The *risk at maturity* is represented by

$$R_m = (\sigma(q_x) \cdot \sqrt{T}) \cdot q_{ex} , \text{ where } T \text{ denotes the number of years.} \quad (23)$$

- The *annualized expected return* is given by

$$E(Return_a) = \frac{q_{forward} - q_{ex}}{T}. \quad (24)$$

- The *annualized Sharpe ratio* is given by

$$S_a = \frac{E(Return_a)}{R_a}. \quad (25)$$

Out of this it is straightforward to solve the equations for the forward rate, i.e.

$$q_{forward} = (1 - T \cdot S_a \cdot \sigma(q_x)) \cdot q_{ex}. \quad (26)$$

The definition of longevity, concerning a 65 year old Austrian male is the same as before; if $x > 81$ the life aged x is a longevity type. Therefore the simulated q-forward contract has a duration of 17 years, i.e. maturity at the age of 81 in the year 2017. To find the fixed rate (q_{ex}^s), anticipated in 2017 at inception by the two counterparties, the *median* of the 5.000 simulated q_{81}^s out of the foregone Monte Carlo Simulation was chosen. The reason therefore is that the median simply divides the simulation in two parts, with each 2.500 rates and equal likelihood. Hence the value for the fixed rate is $q_{ex}^s = 0.1116$ as shown in Figure 36.

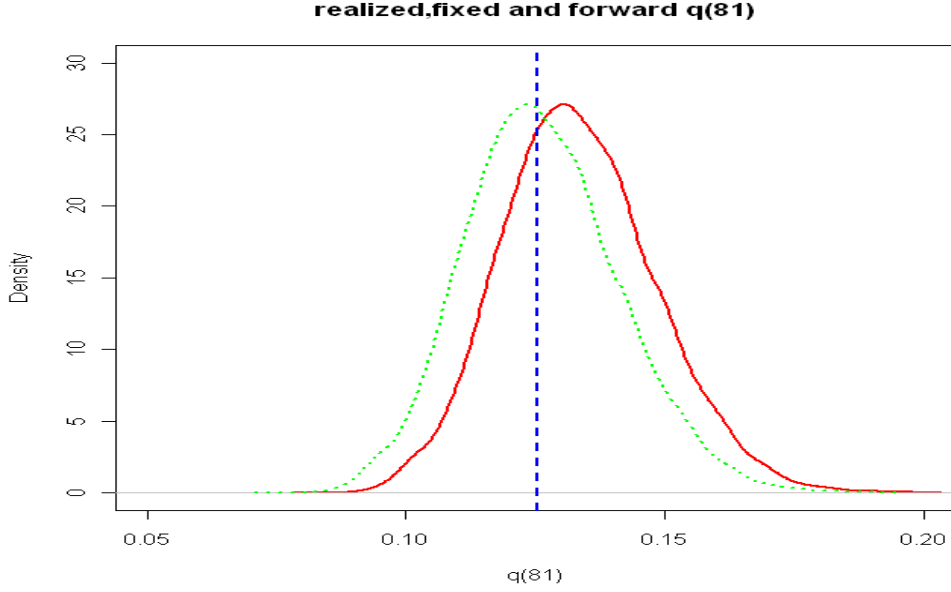


Figure 36: *Realized* (green - dots), *fixed* (blue - dashed) and *forward rate* (red) of $q(81)$. Own calculation.

The volatility $\sigma(q_x)$ of the q_{81} from $t = 1947, \dots, 2003$ is 0.0225, leading to a R_a of 0.0025 and to a R_m of 0.0020. As the S_a is set to 0.25 the derived $q_{forward}^s$ is equal to 0.1179, hence leading to a termpremium (tp), i.e. $tp = q_{forward}^s - q_{ex}^s$, of 0.0062. That is, the $q_{forward}^s$ needs to be around 0.6% above the expected future mortality q_{ex}^s (11.2%) to produce an expected return to risk of 0.25 for JP-Morgan, see Figure 36. Keeping this course of action in plain words, the hedge will pay out to the annuity provider an amount that increases as mortality rates fall to offset the correspondingly higher value of pension liabilities. So, a annuity provider wishing to hedge longevity risk would receive fixed (and pay realized) mortality rates in a q-forward contract, see (Coughlan et al., 2007). Figure 36 suggests that the annuity provider receives at maturity the $not \times 100 \times q_{ex}^s$, i.e. the median of q_{81}^s . JP-Morgan instead gets from the annuity provider the $not \times 100 \times (q_{81}^s + tp)$, i.e. the realized mortality rate in 2017 plus the termpremium, see (Figure 9). The notional (not) is like the annuity set to 1 in this analysis, it could be seen as leverage to fit the payments to the capital at risk. The *net settlement*

payment (nsp) is thus just the difference between

$$nsp = (not \times 100 \times q_{ex}^s) - (not \times 100 \times (q_{81}^s + tp)). \quad (27)$$

Figure 37 shows the possible cashflow streams at maturity, including a 95% confidence interval (dashed lines), i.e. $[-3.7094, 2.1275]$. The outcome of the nsp calculation is rather disappointing as only 1554 $nsp > 0$ and 3446 $nsp < 0$.

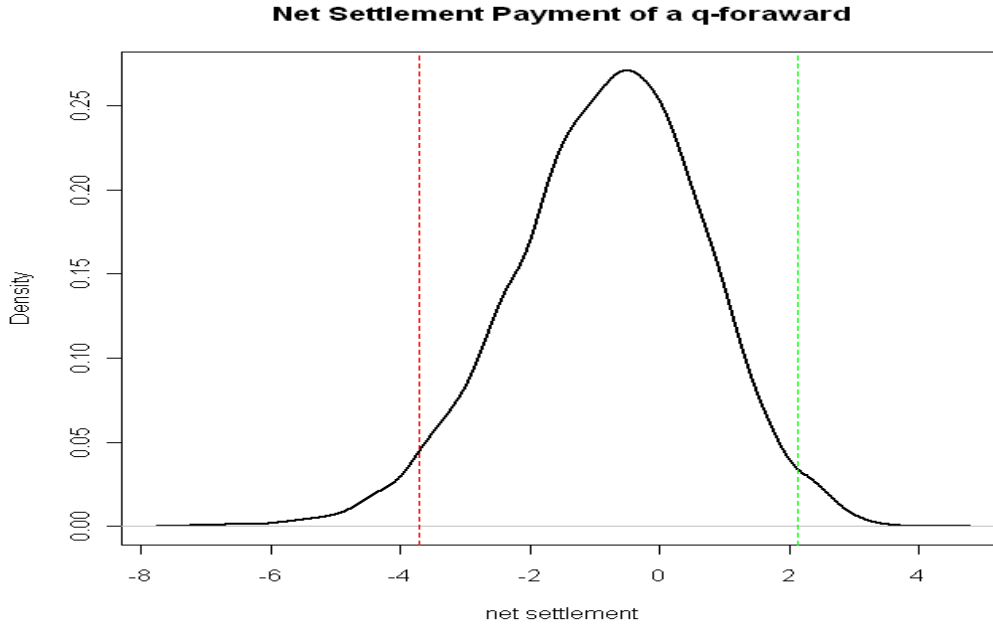


Figure 37: Anticipated Net Settlement Payment of a q-forward for Austrian males aged 81 in 2017. Own calculation.

6.2 Simulation of the $E(NPV)$

To simulate the development of the $E_h(NPV^{forward})$ the same conditional probability (${}_k p_x^h$) as in the foregoing EIB-bond analysis was included. The $E_h(NPV^{forward})$ definition is here a little bit changed because there exists no continuous coupon stream. Thus the q-forward is a zero coupon swap,

i.e. one payment at maturity. However, the $E_h(NPV^{forward})$ is defined as the $E_h(NPV)$ plus the discounted *net settlement payment* (nsp) at $k = 17$,

$$E_h(NPV^{forward}) = \left(\sum_{k=0}^{45} v^k \cdot 1 \cdot p_x^h \right) + v^{17} \cdot nsp, \quad h = 0, \dots, 45. \quad (28)$$

The graphics 38 to 39 include for every scenario h a boxplot of the $E_h(NPV)$ on the left hand side and a boxplot of the $E_h(NPV^{forward})$ on the right hand side. The mean (blue), min (red) and max(green) values of the $E_h(NPV)$ are included as dashed lines.

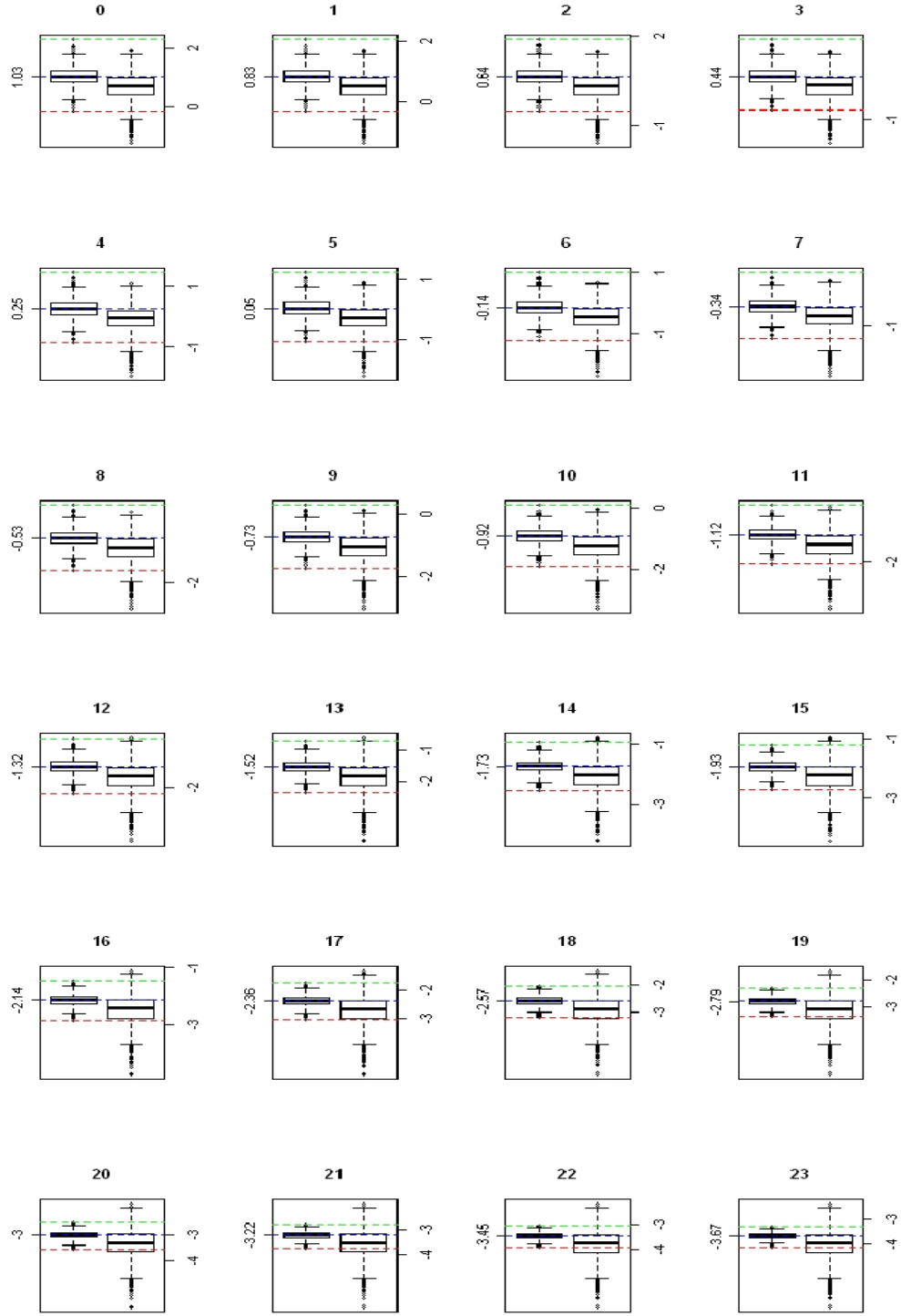


Figure 38: Boxplots of the $E_h(NPV)$, $h = 0, \dots, 23$, left of each picture. Compared to $E_h(NPV^{forward})$, $h = 0, \dots, 23$, right of each picture. $Mean(blue)$, $Min(red)$ and $Max(green)$ values of $E_h(NPV)$ included.

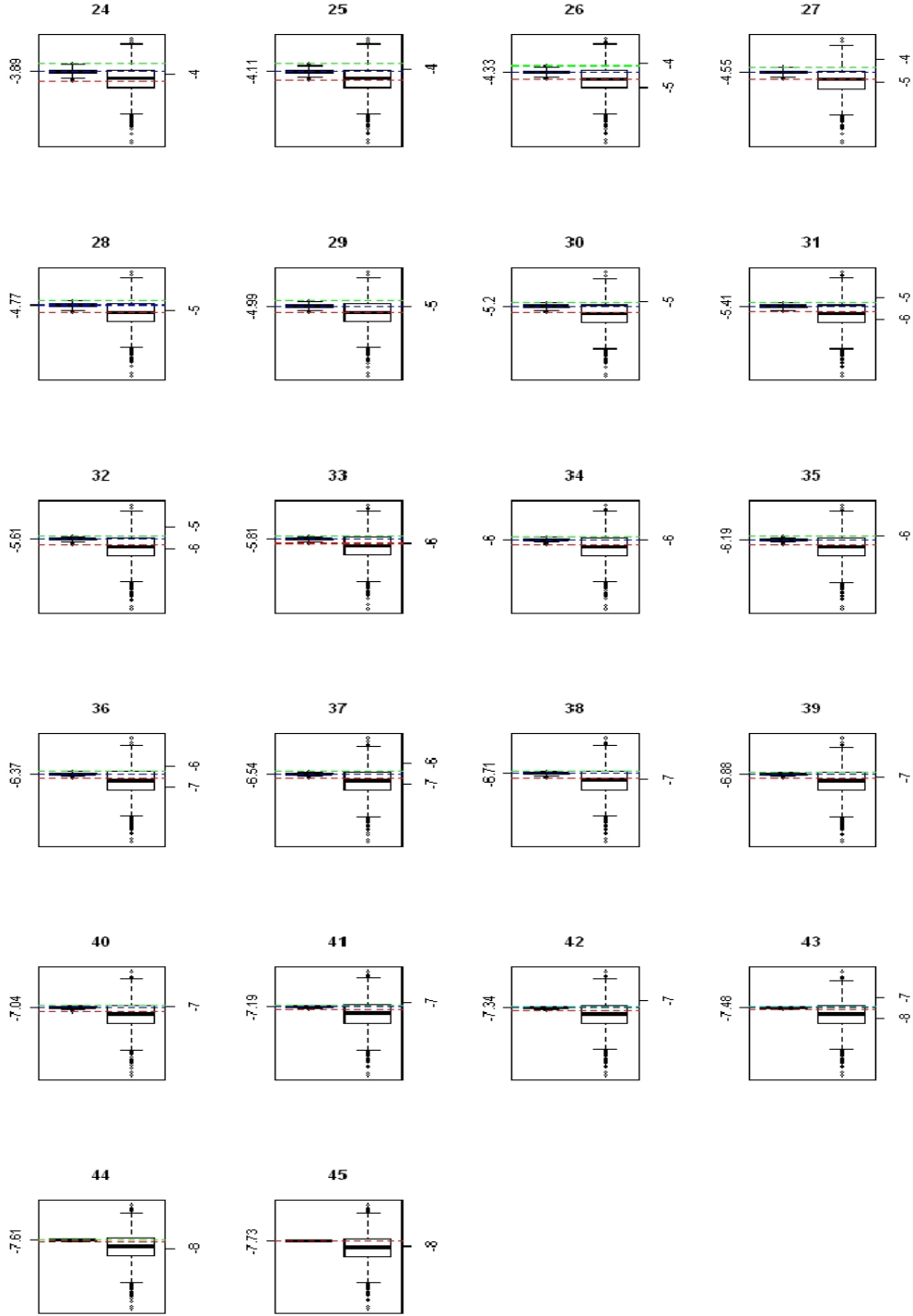


Figure 39: Boxplots of the $E_h(NPV)$, $h = 24, \dots, 45$, left of each picture. Compared to $E_h(NPV^{forward})$, $h = 0, \dots, 23$, right of each picture. $Mean(blue)$, $Min(red)$ and $Max(green)$ values of $E_h(NPV)$ included.

The results of the simulated longevity risk hedge through a q-forward are rather disappointing. The boxplots suggests that every mean of $E_h(NPV^{forward})$ for each scenario h is well below the corresponding mean of $E_h(NPV)$. It is only possible to hedge peak risk, i.e. a very sharp decline in q_x , as the outliers in the boxplots show. In fact there are about 25% of the simulated $E_h(NPV^{forward})$ bigger than the $E_h(NPV)$, for each h . Using the mean as decision criteria, the values for each scenario h are concluded in graphic 40.

	h=0	h=1	h=2	h=3	h=4	h=5	h=6	h=7	h=8	h=9	h=10	h=11
$E_h(NPV)$	1,03	0,83	0,64	0,44	0,25	0,05	-0,14	-0,34	-0,53	-0,73	-0,93	-1,12
$E_h(NPV^{forward})$	0,69	0,50	0,30	0,11	-0,09	-0,29	-0,48	-0,68	-0,87	-1,07	-1,26	-1,46

	h=12	h=13	h=14	h=15	h=16	h=17	h=18	h=19	h=20	h=21	h=22	h=23
$E_h(NPV)$	-1,32	-1,53	-1,73	-1,94	-2,15	-2,36	-2,57	-2,79	-3,01	-3,23	-3,45	-3,67
$E_h(NPV^{forward})$	-1,66	-1,86	-2,07	-2,28	-2,49	-2,70	-2,91	-3,13	-3,35	-3,57	-3,79	-4,01

	h=24	h=25	h=26	h=27	h=28	h=29	h=30	h=31	h=32	h=33	h=34
$E_h(NPV)$	-3,89	-4,12	-4,34	-4,56	-4,77	-4,99	-5,20	-5,41	-5,61	-5,81	-6,00
$E_h(NPV^{forward})$	-4,23	-4,45	-4,68	-4,90	-5,11	-5,33	-5,54	-5,75	-5,95	-6,15	-6,34

	h=35	h=36	h=37	h=38	h=39	h=40	h=41	h=42	h=43	h=44	h=45
$E_h(NPV)$	-6,19	-6,37	-6,55	-6,72	-6,88	-7,04	-7,19	-7,34	-7,48	-7,62	-7,73
$E_h(NPV^{forward})$	-6,53	-6,71	-6,89	-7,05	-7,22	-7,38	-7,53	-7,68	-7,82	-7,95	-8,07

Figure 40: Ranking of the $E(NPV)$ means for each scenario h , with and without q-forward.

The boxplots and the ranking of the means suggests that the term premium (tp), i.e. JP-Morgans profit, is too high, making the product unattractive for hedgers. The fixed rate is negotiated by the two counterparties, thus there is no leeway as both parties try to enforce their own interests. The remaining question is the fair value of the term premium. The fair value of the term premium is the equilibrium of equation (17), i.e. $nsp = 0$, for given q_{81}^s and q_{ex}^s . If $nsp = 0$ there exists no allocation of the value of tp such that at least one is better off without making any other worse off, thus the hedge would be "Pareto Efficient".

The q_{ex}^s are fixed values, so the fair value of the term premium depends on the values of q_{81}^s . In fact every single realization of q_{81}^s , $s = 1, \dots, 5000$ requires a corresponding tp to set $nsp = 0$. Obviously JP-Morgan accepts only positive values of tp . The different densities are plotted in Figure 41. The second and fourth plot from the top shows that the fair value of the term premium decreases with increasing q_{81}^s , and vice versa. The term premium used in this analysis was set at 0.0062. This would be the fair value if the q_{81}^s realizes at 0.1053.

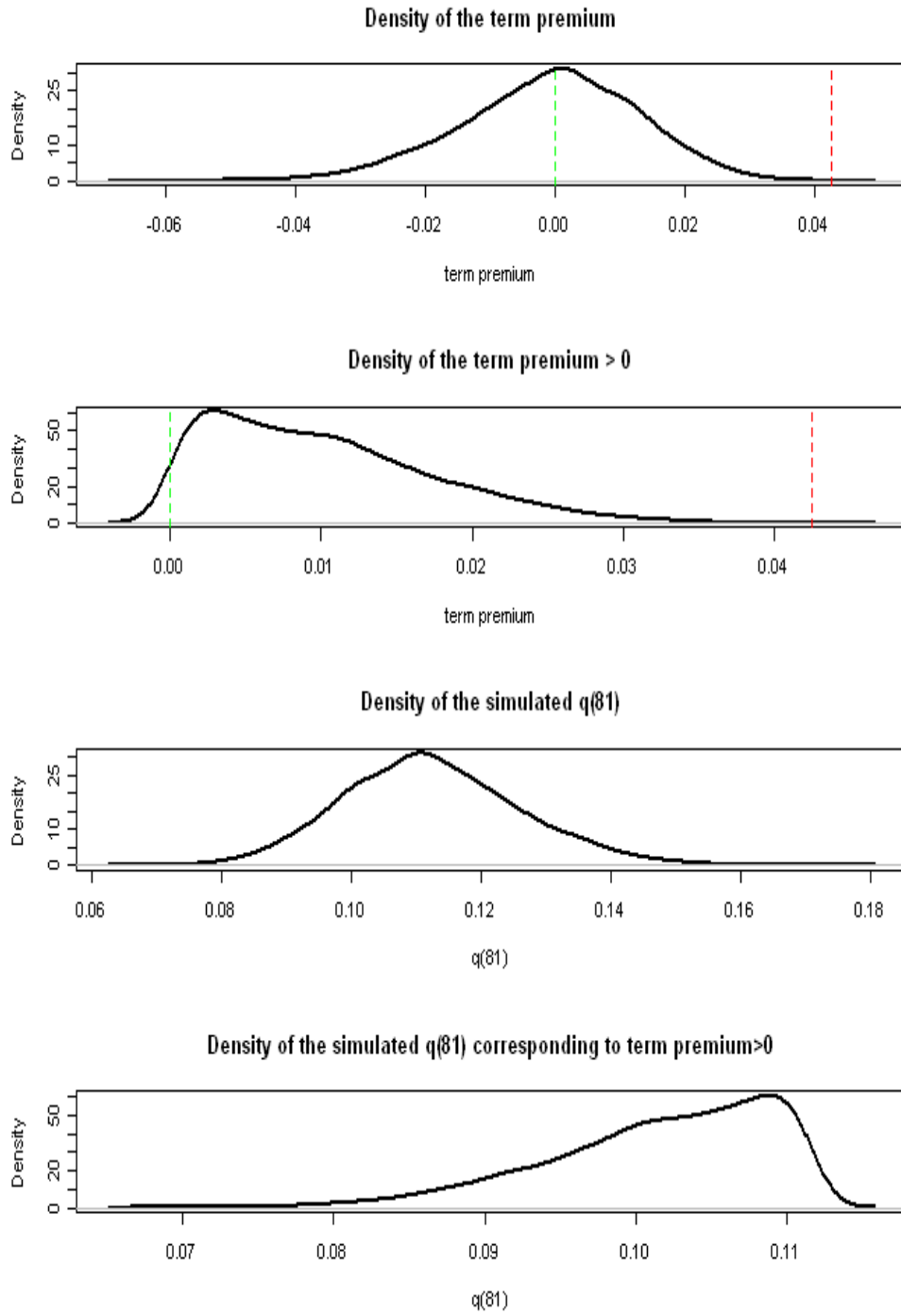


Figure 41: Densities of the term premium and the corresponding q_{81}^s

7 Conclusion

The two variants of mortality linked securities examined in this thesis shows quite different results. Both instruments have their advantages and disadvantages which are concluded below.

7.1 Concerning the EIB-bond the analysis suggests:

7.1.1 Advantages

- The $E_h(NPV)$ improves with respect to the Initial Payment ($IP = 0, \dots, 12$; the price for the bond) for every scenario $h = 0, \dots, 45$ as shown in the ranking of the means, Figure 27 and the boxplot graphics, see Figure 28 up to 35; thus hedging longevity risk is possible, at least for 65 year old Austrian males.
- The coupon stream is not correlated with other asset classes. Bonds are usually correlated with the interest rate or shares or vice versa. But longevity is independent of this asset classes
- The coupon stream delivers a yearly capital income until maturity like usual bond constructions. The simulated coupon streams are shown in Figure 21.
- If one hedges a portfolio of annuity contracts some people will die before the bonds maturity. But once the bond is signed the coupons are guaranteed until maturity and create thus additional capital inflow.
- One should keep in mind that the bond was originally designed for British and Welsh males and not for Austrian males. It was sentenced as too expensive by Biffis and Blake (2001). For Austria this statement is not guilty, as the Expected Net Present Value analysis suggests. Buying 12 units of the bond, $IP = 12$, delivers for all scenarios better results, see Section 5.6.3..

7.1.2 Disadvantages

- Buying a portfolio of bonds is capital intensive. Nearly the whole capital inflow must be paid for a bond to get the best hedging results. This could affect liquidity of the insurance in the short term.
- The 25 year horizon provides probably too less capital inflows to hedge longevity above one hundred and beyond, this should be rethought in further constructions.
- The adverse selection was not included in the index construction, the mortality index used was based on aggregated data. Obviously, people signing annuity contracts, believe to become very old. Thus the results should be interpreted with care, as the index probably underestimates the mortality rates of the annuity stock. Thus every insurer should add a individual security level to the index, based on the longevity improvements in his own stock of annuity contracts.
- The single age cohort of only 65 year old males is too less to hedge an annuity stock. Every annuity stock consists of several individuals, thus there must be a broader offer for ages and of course gender.

Overall one can argue that the disadvantages of this very first version of a longevity bond was not able to reject it as hedging tool under the assumptions of this analysis. The analysis shows that the primary goal, to lift the $E(NPV)$ in higher ages, is possible, at least for 65 year old Austrian males.

7.2 Concerning JP-Morgans q-forward the analysis suggests:

7.2.1 Advantages

- It is easier to include different age cohorts, because there are only two counterparties negotiating and no capital flow is required at inception.

- The maturity is more flexible, i.e. one age group more than one maturity dates. This should probably reduce the costs, i.e. the term premium.
- If people die before maturity, the negative NSP (if q_x increases) is offset, because the liabilities for the involved contracts are reduced.
- The underlying mortality rates are easier to agree because of the bilateralism. For instance official mortality data or mortality data out of the annuity stock.
- Presence of asymmetric information could be a advantage for the holder of longevity exposure, because he knows more about his annuity stock

7.2.2 Disadvantages

- Under the ruling assumptions of this analysis, the q-forward failed. The mean of the $E(NPV)$ was lower as without hedge. However there was some outliers suggesting that only the hedge of peak risk is possible, i.e. a very sharp decline in q_x . But this is rather implausible, as the sampling of the rates suggests, see Figure 20.
- The outcome depends on the counterparties agreement of the fixed rate at inception. Hence the model used to forecast is from importance. At the moment there is no commonly accepted model for determining expectations about mortality improvements over time, see Biffis and Blake(2009).
- The term premium (tp) , i.e. the price, is the crucial parameter. The fair value of the term premium could only be calculated if the realized future mortality rate is known.

The hedge through a q-forward depends strongly on the agreements at inception, thus it is difficult to analyze without an concrete underlying portfolio. But the foregone analysis gives an general overview how this instrument works and what the important points of a q-forward contract for an annuity

provider are. One publicly announced deal was for instance between Canada Life²⁰ and JP Morgan with an value of £500m and a duration of 40 years in October 2008.

²⁰Life&Pensions Oct.2008

A Mortality tables

In insurance mathematics the basis for further calculations is the corresponding mortality table. The numbers collected by the census are out of an open population hence they are impacted by migration. Such databases deliver so called crude rates because the data are unadjusted. This mortality tables shows for younger ages a random variability because of missing data. However this data are often smoothed across ages to reduce the influence of outliers, the rates are then called graduated mortality rates. For Austria the mortality table 2000/2002 is the last graduated one out of the population census (see Statistik Austria, Demographische Maßzahlen).

The key variables of mortality tables are²¹:

- Survivors in the age of x : $l(x)$
- Number of deaths in $[x, x + 1]$: $d(x) = l(x) - l(x + 1)$
- Death probability in $[x, x + 1]$: $q(x) = d(x)/l(x)$
- Average Survivors in $[x, x + 1]$: $L(x) = (l(x) + l(x + 1))/2$
- Aggregated residual lifetime of a cohort in the age of x : $T(x) = L(x) + L(x + 1) + \dots + L(100)$
- Average residual lifetime in the age of x : $e(x) = T(x)/l(X)$.

²¹<http://www.lebenserwartung.info/index-Dateien/sterbetafel.htm>

B R-Code

```
1: #read austrian data
2: library(demography)
3: library(xlsReadWrite)
4: library(Hmisc)
5: library(financial)
6: setwd("C:/Dokumente und Einstellungen/Roger/Desktop/Final Code")
7: qx<-read.demogdata("rates.txt", "population.txt",type="mortality" ,
8: label="Austria", skip=2,popskip=2,max.mx = 10,0)
9: #plot total rates
10: plot(qx,series="male",xlim=c(0,115),ylim=c(-11,2))
11: axis(1,at=(seq(0,110,10)))
12: axis(2,at=(seq(-10,3,1)))
13: abline(h=0,col="darkgrey",lty=2)
14: abline(v=100,col="darkgrey",lty=2)
15: legend("topleft",c("red = 1947","violet = 2008"),
16: text.col=c("red","violet"),bty="n")
17:
18: ##
19: # smooth demogdata
20: #variance of raw and smoothed data for year=2000
21: #raw
22: va<-extract.years(qx,2008)
23: na<-which(is.na(va$rate$male))
24: va$rate$male[na]<-0
25: va<-va$rate$male
26: var(va[95:110])*100
27: #smooth
28: qx<-smooth.demogdata(qx,method="mspline")
29: #smoothed data
30: va2<-extract.years(qx,2008)
31: na2<-which(is.na(va2$rate$male))
```

```

32: va2$rate$male[na2]<-0
33: va2<-va2$rate$male
34: var(va2[95:110])*100
35:
36: #extract age groups
37: #mortality modeling of 65 year old austrian males - verify the usefulness of
38: #the EIB bond
39:
40: # 1) TEST THE ACCURACY OF FORECAST THROUGH MSE & ISE, CHOOSE THE
ORDER K OF THE
41: # MODELL
42: iter<-(1:10)
43: qx.isesum<-0
44: qx.msesum<-0
45: for (i in iter) {
46: qx.test<-extract.years(qx,years=1947:1983)
47: qx.fit<-fdm(qx.test,order=i,series="male",method="M",lambda=3)
48: qx.error<-compare.demogdata(qx,forecast(qx.fit,25),interpolate=TRUE)
49: qx.isesum[[i]]<-sum(qx.error$int.error[,3])
50: qx.msesum[[i]]<-sum(qx.error$mean.error[,3]) }
51: qx.isesum
52: qx.msesum
53:
54: # errors are lowest with order 6, this leads to the following modell
55: # for males
56: #forecast whole sample
57: # FOR 25 years, i.e. the duration of the EIB bond!!!
58: qx.eib<-extract.years(qx,years=1947:2002)
59: qx.fdm<-fdm(qx.eib,series="male",order=6,method="M",lambda=3,max.age=110)
60:
61: #explained variance
62: qx.fdm$varprop
63: qx.fdm$varprop*100

```

```

64: #forecast ets
65: qx.fcst<-forecast(qx.fdm,h=25,method="ets")
66: qx.sim<-simulate(qx.fcst,nsim=5000)
67: #read qx -> diagonale start with 65 years
68: # ACHTUNG!!!! nsim == nrow == sim == iter !!!!!!!!!!!
69: simvalues <- matrix(nrow = 5000, ncol = 25)
70: ro<-66:90
71: colu<-1:25
72: male65<-NULL
73: sim<-1:5000
74: meansimu<-NULL
Tinn-R - [C:\Dokumente und Einstellungen\Roger\Desktop\Final Code\final code.r] 2/8
75: mediansimu<-NULL
76: for ( si in sim) {
77: for (co in colu) for (r in ro) { if (r-co==65) male65[co]
78: <-qx.sim[r,co,si]}
79: simvalues[si,]<-male65 }
80: for (co in colu) meansimu[co]<-mean(simvalues[,co])
81:
82: write.xls(simvalues,file="simvalues",sheet=1,from=1)
83: iter<-1:5000
84: period<-2003:2027
85: plot(period,simvalues[1,],xaxt="n",type="l",ylim=c(0.000,0.35),
86: ylab="qx(65,2003) up to qx(89,2027)",xlab="Year")
87: legend("topleft",c("mean","actual values"),cex=0.8,fill=c("red","black"),
88: col=c("red","black"))
89: title("Simulation of qx (Austrian males aged 65-89)")
90: for (i in iter) lines(period,simvalues[i,],col=i,type="l")
91: lines(period,meansimu,type="l",col="red",lwd=3.5)
92: #include actual values in graphic
93: qx.actual<-diag(qx$rate$male[66:71,57:62])
94: per<-2003:2008
95: points(per,qx.actual,col="black",pch=19,lwd=0.1)

```

```

96: axis(1,at=seq(2003,2027,1))
97: # histogram of simulated rates
98: d<-(density(simvalues[,25]))
99: hist(simvalues[,25],freq=FALSE)
100: lines(d, col="red")
101:
102: ##
103: #Payoff/ Cashflow Simulation
104: #
105: #COUPONS
106: indexvalues<- read.xls("C:\\Dokumente und Einstellungen\\Roger\\Desktop\\
107: Final Code\\indexvalues.xls")
108: initial<-50000000
109: coupons <- matrix(nrow = 5000, ncol = 25)
110: iter<-1:25
111: simul<-1:5000
112: coupon<-NULL
113: for (j in simul) { for (i in iter) (coupon[i]<-indexvalues[j,i]*initial)
114: coupons[j,]<-coupon}
115: write.xls(coupons,file="coupons",sheet=1,from=1)
116:
117: #mean Coupons
118: meancoupons<-NULL
119: colu<-1:25
120: for (co in colu) meancoupons[co]<-mean(coupons[,co])
121: #PLOT Coupons
122: iter<-1:5000
123: period<-2003:2027
124: plot(period,coupons[1,],xaxt="n",type="l", ylim=c(2000000,50000000),
125: ylab="Coupons",xlab="Year")
126: legend("topright",c("mean","actual values"),cex=0.8,fill=c("red","black"),
127: col=c("red","black"))
128: title("EIB/ Bond - Coupon Simulation - in million Pounds")

```

```

129: for (i in iter) lines(period,coupons[i,],col=i,type="l")
130: lines(period,meancash,type="l",col="red",lwd=3.5)
131: per<-2003:2008
132: points(per,act*50000000,col="black",pch=19,lwd=0.1)
133: axis(1,at=seq(2003,2027,1))
134:
135:
136: # 95% KI Mean forecast +/- 2*SD
137: KIo<-mean(coupons[,d]) + 1.96*sqrt(var(coupons[,d]))
138: KIu<-mean(coupons[,d]) - 1.96*sqrt(var(coupons[,d]))
139: abline(v=KIo,col="green",lwd="1.8",lty=2)
140: abline(v=KIu,col="red",lwd="1.8",lty=2)
141: axis(1,at=c(mean(coupons[,d]),KIo,KIu)) }
142:
143: ##
144: #coupons if one buys for 1,2,3,...,11,12
145:
146: coupons<- read.xls("C:\\Dokumente und Einstellungen\\Roger\\Desktop\\mortality\\
147: coupons.xls")
148: unitcoupons<-(coupons/540000000)*12.6052047902006 #coupons if one buys
Tinn-R - [C:\\Dokumente und Einstellungen\\Roger\\Desktop\\Final Code\\final code.r] 3/8
149: for one unit
150: simu<-1:5000
151: notional<-12.6052047902006
152: irr<-0.0444
153: year<-1:25
154: discounted<-matrix(nrow = 5000, ncol = 25)
155: unitnpv<-vector( length=5000)
156: v<-NULL
157:
158: for (si in simu) { for (y in year) v[y]<-unitcoupons[si,y]/((1+irr)^y)
159: discounted[si,]<-v
160: unitnpv[si]<-notional-sum(discounted[si,])}

```

```

161: #mean
162: meanunitnpv<-mean(unitnpv)
163: sigma<-sqrt((sum(unitnpv^2)/10000 - mean(unitnpv)^2))
164: KLu<-qnorm(0.025,mean(unitnpv),sigma)
165: KLo<-qnorm(0.975,mean(unitnpv),sigma)
166: ##
167: #buying simu of npv
168: #plot(npv,k_pi_x, col="red",lwd="2")
169: #densities
170: par(mfrow=c(4,3))
171: iter<-1:12
172: for (i in iter) { plot(npv,k_pi_x, col="orange",type="l",ylab="probabilities")
173: unitcoupons<-(coupons/540000000)*i
174: ro<-1:5000
175: jter<-2:45
176: b<-matrix(nrow=1,ncol=45)
177: pension<-1
178: npv2<-matrix(nrow=5000,ncol=45)
179: zer<-matrix(c(0:0),nrow=5000,ncol=20)
180: unitcoupons<-cbind(unitcoupons,zer)
181: for (r in ro ) { b[1]<-12.6052047902006-i
182: for ( j in jter)
183: (b[j]<-(b[j-1]-pension*v[j]+unitcoupons[r,j]*v[j]))
184: npv2[r,]<-b }
185: for (r in ro){
186: lines(npv2[r,],k_pi_x[1:45],col=i,type="l")}
187: lines(npv,k_pi_x, col="orange",lwd="2",type="l")}
188:
189: #lines
190: period<-1:45
191: par(mfrow=c(4,3))
192: iter<-1:12
193: for (i in iter) { plot(period,npv[1:45], col="orange",type="l",ylab="npv")

```

```

194: unitcoupons<-(coupons/540000000)*i
195: ro<-1:5000
196: jter<-2:45
197: b<-matrix(nrow=1,ncol=45)
198: pension<-1
199: npv2<-matrix(nrow=5000,ncol=45)
200: zer<-matrix(c(0:0),nrow=5000,ncol=20)
201: unitcoupons<-cbind(unitcoupons,zer)
202: for (r in ro ) { b[1]<-12.6052047902006-i
203: for ( j in jter)
204: (b[j]<-(b[j-1]-pension*v[j]+unitcoupons[r,j]*v[j]))
205: npv2[r,]<-b }
206: for (r in ro){
207: lines(period[1:45],npv2[r,],col=i,type="l")}
208: lines(period[1:45],npv[1:45], col="orange",lwd="2",type="l")}
216: #buying simu of irr
217: coupons<- read.xls("C:\\Dokumente und Einstellungen\\Roger\\Desktop\\mortality\\
218: coupons.xls")
219:
220: zer<-matrix(c(0:0),nrow=10000,ncol=20)
221: unitcoupons<-cbind(as.matrix(unitcoupons),zer)
222: simu<-1:10000
223: jter<-1:12
Tinn-R - [C:\\Dokumente und Einstellungen\\Roger\\Desktop\\Final Code\\final code.r] 4/8
224: b<-matrix(nrow=1,ncol=45)
225: irr<-matrix(nrow=10000,ncol=12)
226: pension<-1
227: for (j in jter) { for (si in simu) {
228: npv0<-function(co,r) {
229: 12.6052047902006-j+sum((co/(1+r)^(seq(along=co)))))-
230: sum((1/(1+r)^(seq(along=co))))}
231: co<-unitcoupons[si,]
232: irr[si,j]<-uniroot(npv0,c(0,1),co=co)$root}}

```



```

234: ##
235: # secondary market rate (all Emittents) 2003 riskless interest rate
236: # MRR = 4.44%
237: # NPV calculation
238: simu<-1:5000
239: notional<-540000000
240: irr<-0.0444
241: year<-1:25
242: discounted<-matrix(nrow = 10000, ncol = 25)
243: npv<-vector( length=5000)
244: v<-NULL
245:
246: for (si in simu) { for (y in year) v[y]<-coupons[si,y]/((1+irr)^y)
247: discounted[si,]<-v
248: npv[si]<-notional-sum(discounted[si,])}
249:
250: length(which(npv<0))
251: qqnorm(npv)
252: qqline(npv)
253: length(which(npv<(-20000000)))
254:
255: #net present values of coupons plot
256: par(mfrow=c(1,1))
257: dnpv<-density(npv)
258: hist(npv,freq=FALSE)
259: lines(dnpv,col="black",lwd=2)
260: #standardabweichung
261: sigma<-sqrt((sum(npv^2)/5000 - mean(npv)^2))
262:
263: KLu<-qnorm(0.025,mean(npv),sigma)
264: KLo<-qnorm(0.975,mean(npv),sigma)
265: abline(v=KLo,col="green",lwd=1.8)
266: abline(v=KLu,col="red",lwd=1.8)

```

```

267: abline(v=mean(npv),col="orange",lwd=1.8)
268: abline(v=0,col="blue",lty=2,lwd=2.2)
269: #positv values in KI
270: length(which(npv>0 & npv<31158720))
271:
272: ##
273: #IRR of EIB
274:
275: simu<-1:5000
276:
277: npv<-function(co,r) {
278: -540000000+sum((co/(1+r)^(seq(along=co))))}
279: for (si in simu) { co<-coupons[si,]
280: irr[si]<-uniroot(npv,c(0,1),co=co)$root}
281:
282: #plot iir density with 95% confidence intervall
283: sigma<-sqrt((sum(irr^2)/10000 - mean(irr)^2)) # mean 0.04235254
284: KIu<-qnorm(0.025,mean(irr),sigma) #0.03701628
285: KIlo<-qnorm(0.975,mean(irr),sigma)#0.04768879
286: den<-density(irr)
287: hist(irr,freq=FALSE,main="density of the IRR",ylim=c(0,160),
288: xlim=c(0.028,0.055))
289: lines(den, col="black",lwd="2")
290: abline(v=KIu,col="red",lwd="1.8")
291: abline(v=KIlo,col="green",lwd="1.8")
292: abline(v=mean(irr),col="orange",lwd="1.8")
293: abline(v=0.0444,col="blue",lwd="1.8",lty=2)
296: ##
297: #qx FOR 46 years 0,...,45
298: qx.eib<-extract.years(qx,years=1947:2002)
Tinn-R - [C:\Dokumente und Einstellungen\Roger\Desktop\Final Code\final code.r] 5/8
299: qx.fdm<-fdm(qx.eib,series="male",method="M",lambda=3,max.age=110)
300: qx.fcst2<-forecast(qx.fdm,h=46,method="ets")

```

```

301: qx.sim2<-simulate(qx.fcst2,nsim=5000)
302: #qx auslesen -> diagonale start mit 65 jahren
303: # ACHTUNG!!!! nsim == nrow == sim == iter !!!!!!!!!!!
304: #ACHTUNG qx only up to 110 years available, AVOE2005R 120 years
305: simvalues2 <- matrix(nrow = 5000, ncol = 46)
306: ro<-65:110
307: colu<-1:46
308: male652<-NULL
309: sim<-1:5000
310: # meansimu<-NULL
311: # mediansimu<-NULL
312: for ( si in sim) {
313: for (co in colu) for (r in ro) { if (r-co==64) male652[co]<-
314: qx.sim2[r,co,si]}
315: simvalues2[si,]<-male652 }
316:
317: write.xls(simvalues2,file="simvalues2",sheet=1,from=1)
318:
319: # set maximum value of qx to 1
320: ro<-1:5000
321: colu<-1:46
322: for ( r in ro) { for (co in colu) if (simvalues2[r,co]>1) simvalues2[r,co]<-1}
323: write.xls(simvalues2,file="simvalues2_1",sheet=1,from=1) #sim2smoothed1->qx<=1,
324: because of outliers in raw data set to 1
327: #slope for px
328: #
329: simvalues2_1<- read.xls("C:\\Dokumente und Einstellungen\\Roger\\Desktop\\Final
330: Code\\simvalues2_1.xls")
331: ro<-1:5000
332: iter<-1:46
333: px<-matrix(nrow=5000,ncol=46)
334: for (i in iter ) {
335: for ( r in ro)

```

```

336: px[r,i]<-1-simvalues2_1[r,i]
337: write.xls(px,file="px",sheet=1,from=1)
338:
339: #slope for kpx
340: #
341: simvalues2_1<- read.xls("C:\\Dokumente und Einstellungen\\Roger\\Desktop\\Final
342: Code\\simvalues2_1.xls")
343: px<- read.xls("C:\\Dokumente und Einstellungen\\Roger\\Desktop\\Final Code\\
344: px.xls")
345: ro<-1:5000
346: iter<-1:45
347: kpx<-matrix(nrow=5000,ncol=46)
348: for (r in ro ) { kpx[r,1]<-px[r,1]
349: for ( i in iter)
350: kpx[r,i+1]<-(kpx[r,i]*px[r,i+1])}
351: write.xls(kpx,file="kpx",sheet=1,from=1)
352:
353: ##
354: #slope for kpix
355: #
356: simvalues2_1<- read.xls("C:\\Dokumente und Einstellungen\\Roger\\Desktop\\Final
357: Code\\simvalues2_1.xls")
358: kpx<- read.xls("C:\\Dokumente und Einstellungen\\Roger\\Desktop\\Final Code\\
359: kpx.xls")
360: ro<-1:5000
361: iter<-1:46
362: kpix<-matrix(nrow=5000,ncol=46)
363: for (r in ro ) { for ( i in iter)
364: kpix[r,i]<-(kpx[r,i]*(simvalues2_1[r,i]))}
365: write.xls(kpix,file="kpix",sheet=1,from=1)
366:
367: ##
368: #The conditional probabilities

```

```

370: kter<-2:46
371: hter<-1:46
372: iter<-1:5000
373: condprob<-matrix(nrow=5000,ncol=46)
Tinn-R - [C:\Dokumente und Einstellungen\Roger\Desktop\Final Code\final code.r] 6/8
374: condprob[,1]<-1
375: #ha is to set manuel
376: for (ha in hter) {
377: for (i in iter) for (k in kter) {
378: if (k<=ha) {condprob[i,k]<-1} else{ condprob[i,k]<-px[i,k]*condprob[i,k-1]}
379: }}
380:
381: write.xls(condprob,file="h_ha",sheet=1,from=1)
382:
383: ###
384: # computing the coupons
385: #
386: simvalues<- read.xls("C:\\Dokumente und Einstellungen\\Roger\\Desktop\\Final
387: Code\\simvalues.xls")
388: coupons <- matrix(nrow = 5000, ncol = 25)
389: sim<-1:5000
390: year<-2:25
391: index<-1
392: for (si in sim){ for (y in year) (index[y]<-index[y-1]*(1-simvalues[si,y]))
393: coupons[si,]<-index}
394: write.xls(coupons,file="coupons",sheet=1,from=1)
395:
396: ###
397: #boxplots
398:
399:
400: iter<-(46:48)
401: par(mfrow=c(3,1))
402: for (i in iter){

```

```

403: x<- read.xls("C:\\Dokumente und Einstellungen\\Roger\\Desktop\\Final Code\\
404: ENPV_bond.xls",sheet=i)
405: colnames(x)<-c(0:12)
406: boxplot(x,yaxt="n",ylim=c(min(x)-0.5,max(x)+0.5))
407: title(main=(i-1))
408: axis(2,at=(round(median(x[,1]),2)))
409: #axis(4,at=(round(min(x[,1]),0)))
410: # axis(4,at=(round(max(x[,1]),0)))
411: abline(h=(median(x[,1])), col="blue",lty=2)
412: abline(h=(max(x[,1])), col="green",lty=2)
413: abline(h=(min(x[,1])), col="red",lty=2)
414: }
415:
416: ##
417: #ranking of the means
419: ranking<- matrix(nrow = 46, ncol = 13)
420: iter<-1:46
421: jter<-1:13
422: for ( i in iter ) {
423: x<- read.xls("C:\\Dokumente und Einstellungen\\Roger\\Desktop\\Final Code\\
424: ENPV_bond.xls",sheet=i)
425: for ( j in jter) { ranking[i,j]<-mean(x[,j]) } }
426:
427: write.xls(ranking,file="ranking",sheet=1,from=1)
431: ##
432: # Q-FORWARD SIMULATION
433:
434:
435: #read austrian data
436: library(demography)
437: library(xlsReadWrite)
438: library(Hmisc)
439: library(financial)

```

```

440: library(sm)

441: setwd("C:/Dokumente und Einstellungen/Roger/Desktop/Final Code")

442: qx<-read.demogdata("rates.txt", "population.txt",type="mortality" ,

443: label="Austria", skip=2,popskip=2,max.mx = 10,0)

444:

445: actualq65<-qx$rate$male[66,57] #actual q65=0.016059

446: sim2smoothed1<- read.xls("C:\\Dokumente und Einstellungen\\Roger\\Desktop\\

447: Final Code\\simvalues2_1.xls")

448: q81<-sim2smoothed1[,21] #simulated q81

Tinn-R - [C:/Dokumente und Einstellungen/Roger/Desktop/Final Code\\final code.r] 7/8

449: mediq81<-median(q81) # mediq81=0.1116578 fixed rate

450: meanq81<-mean(q81) # meanq81=0.1124574

451: minq81<-min(q81) # minq81=0.06917058

452: maxq81<-max(q81) # maxq81=0.1738903

453: avoeq81<-0.054634909756875

454: d81<-density(q81)

455: plot(hist(q81),freq=FALSE,xlab="q(81)",main="Simulated q(81)",xaxt="n",

456: ylim=c(0,30),xlim=c(0.05,0.20))

457: lines(density(q81),lwd=2,col="red")

458: axis(1,seq(0.05,0.20,0.025))

459: abline(v=mediq81,col="blue",lty=2,lwd=2)

460:

461: #volatility of q81

462: q81rates<-qx$rate$male[82,1:57] #actual rates over time

463: volq81<-sd(q81rates) #0.02251644

464: annualrisk<-volq81*mediq81 #0.002514136

465: gena_vola<-volq81*sqrt(16)

466: risk_maturity<-volq81*gena_vola #0.00202796

467: annual_sharpe_ratio<-0.25

468: forwardrate<-(annual_sharpe_ratio*volq81*10+1)*mediq81 #0.1179431

469: termpremium<-forwardrate-mediq81 #0.00628534

470: discount<-1-mediq81/forwardrate # 0.05329127

471:

```

```

472: #percentage changes of mortality from q65 to 5000 simulatedq75
473: perc<-vector(length=5000)
474: iter<-1:5000
475: for (i in iter) { perc[i]<-q81[i]/actualq65*100}
476: percd<-density(perc)
477: sigma<-sqrt((sum(perc^2)/5000 - mean(perc)^2))
478: KLu<-qnorm(0.025,mean(perc),sigma)
479: KLo<-qnorm(0.975,mean(perc),sigma)
480: plot(percd,lwd=2)
481: abline(v=KLu,col="green",lty=3)
482: abline(v=KLo,col="darkred",lty=3)
483: abline(v=mediq81/actualq65*100,col="blue",lwd=2)
484: abline(v=forwardrate/actualq65*100,col="red",lty=2,lwd=2)
485:
486:
487: #net settlement if expected is the median
488: netset<-matrix(nrow=5000,ncol=1)
489: jter<-1 # notional = 1
490: iter<-1:5000
491: for (j in jter){
492: for ( i in iter)
493: netset[i,j]<-(j*mediq81*100-j*(q81[i]+termpremium)*100)
494: }
495: write.xls(netset,file="netset",sheet=1,from=1)
496: length(which(netset>0)) #1554
497: length(which(netset<0)) #3446
498: length(which(netset>-3.709404&netset<0)) #3344
499:
500: ##
501: #fair value
502: ##**
503: qx<-read.demogdata("rates.txt", "population.txt",type="mortality" ,
504: label="Austria", skip=2,popskip=2,max.mx = 10,0)

```



```

505: actualq65<-qx$rate$male[66,57] #actual q65=0.016059
506: sim2smoothed1<- read.xls("C:\\Dokumente und Einstellungen\\Roger\\Desktop\\
507: Final Code\\simvalues2_1.xls")
508: q81<-sim2smoothed1[,21] #simulated q81
509: mediq81<-median(q81) # mediq81=0.1116578 expected rate
510: tp<-vector(length=5000) # term premium vector
511: qex<-vector(length=5000)
512: qex[1:5000]<-mediq81 #expected rate vector
513: q81<-as.vector(q81) # simulated rate vector
514: tp<-qex-q81
515: length(which(tp>0)) #2500
516: par(mfrow=c(4,1))
517: d<-density(tp)
518: plot(d,col="black",xlab="term premium",main="Density of the term
519: premium",lwd=2)
520: abline(v=0,col="green",lwd=1.8,lty=2)
521: abline(v=max(tp),col="red",lwd=1.8,lty=2)
522: tp2<-tp[tp>0] #positiv tp
523: d<-density(tp2)
Tinn-R - [C:\\Dokumente und Einstellungen\\Roger\\Desktop\\Final Code\\final code.r] 8/8
524: plot(d,col="black",xlab="term premium",main="Density of the term
525: premium > 0",lwd=2)
526: abline(v=0,col="green",lwd=1.8,lty=2)
527: abline(v=max(tp),col="red",lwd=1.8,lty=2)
528: plot(density(q81),col="black",xlab="q(81)",
529: main="Density of the simulated q(81)",lwd=2)
530: c<-which(tp>0)
531: a<-q81[c]
532: plot(density(a),col="black",xlab="q(81)",
533: main="Density of the simulated q(81)
534: corresponding to term premium>0",lwd=2)
535: ##
536: #plot realized, fixed and forward

```

```

537: ##**
538: par(mfrow=c(1,1))
539: plot(density(q81+termpremium),col="red",xlab="q(81)",main="realized,fixed and
540: forward q(81)",ylim=c(0,30),xlim=c(0.05,0.2),lwd=2)
541: lines(density(q81),col="green",lwd=2,lty=3)
542: abline(v=mediq81,col="blue",lwd=2,lty=2)
543: #plot net settlement payment of a q-forward
544: plot(density(netset[,1]),col="black",lwd=2,xlab="net settlement",main="Net
545: Settlement Payment of a q-foraward")
546: abline(v=qnorm(0.025,mean(netset[,1]),sd(netset[,1])),col="red",
547: lty=2) #-3.709404
548: abline(v=qnorm(0.975,mean(netset[,1]),sd(netset[,1])),col="green",
549: lty=2) # 2.127586
550:
551: ##
552: #boxplots
553: #*
554:
555: enpvnspl<- read.xls("C:\\Dokumente und Einstellungen\\Roger\\Desktop\\
556: Final Code\\ENPV+NSP.xls",sheet=3)
557: odd<- read.xls("C:\\Dokumente und Einstellungen\\Roger\\Desktop\\
558: Final Code\\ENPV+NSP.xls",sheet=4)
559: iter<-as.vector(odd[37:46,1])
560: par(mfrow=c(3,4))
561: for (i in iter){
562: boxplot(cbind(enpvnspl[,i],enpvnspl[,i+1]),yaxt="n",
563: xaxt="n")#ylim=c(min(enpvnspl)-0.5,max(enpvnspl)+0.5))
564: title(main=odd[i,2])
565: axis(2,at=(round(median(enpvnspl[,i]),2)))
566: axis(4,at=(round(min(enpvnspl[,i]),0)))
567: axis(4,at=(round(max(enpvnspl[,i]),0)))
568: abline(h=(median(enpvnspl[,i])), col="blue",lty=2)
569: abline(h=(max(enpvnspl[,i])), col="green",lty=2)

```

```
570: abline(h=(min(enpvnspl,i)), col="red",lty=2)
```

C Abstract-German

Die ex post unterschätzte Entwicklung der Sterblichkeitsraten bringt für die Pensionsfonds und Rentenversicherer, im privaten wie im öffentlichen Bereich, eine zunehmende Kapitalintensivierung mit sich. Die negativen Auswirkungen auf die Bilanzen und die Liquidität der Unternehmen sowie des Staatshaushaltes regten in der vergangenen Dekade die Suche nach Auswegen an. Blake und Burrows (2001) waren die ersten die einen Transfer des Langlebigkeitsrisikos in Richtung Kapitalmarkt befürworteten. Investmentbanken und Versicherer konstruierten daraufhin die ersten Finanzinstrumente um das Langlebigkeitsrisiko zu hedgen. In dieser Diplomarbeit sollen die verschiedenen Instrumente aufgezeigt werden und durch stochastische Modellierung der möglichen zukünftigen Sterblichkeitsraten, von 65 jährigen Österreichern, auf ihre Tauglichkeit als Hedge Instrument getestet werden. Als Entscheidungskriterium wurde der Net Present Value herangezogen welcher mit einer bedingten Überlebenswahrscheinlichkeit gewichtet wurde, i.e ein sogenannter Expected Net Present Value. Die Analyse zeigte, dass ein hedge des Langlebigkeitsrisikos mit der Anleihe der Europäischen Investmentbank aus 2003 möglich gewesen wäre, zumindest für 65 jährige Österreicher. Die Expected Net Present Values waren in allen Szenarien durch den Kauf der Anleihe höher als jene ohne hedge. Als zweite Möglichkeit wurde ein zero coupon swap, JP Morgans q-forward, herangezogen. Diese Analyse brachte enttäuschende Ergebnisse mit sich. Unter den Annahmen dieser Arbeit, ergab die Analyse, dass der q-forward zu teuer gewesen wäre. Die Expected Net Present Values waren alle niedriger, als jene ohne hedge.

D Abstract

The ex-post underestimated development of mortality rates leads to a significant increase of the liabilities for pension funds and annuity providers. The overall negative effects on the balance sheet and thus the liquidity of enterprises and governments stressed in past decades the search for back door solutions. Blake and Burrows (2001) was the first to propose a transfer

of longevity risk to capital markets. Investmentbanks and Insurers followed this idea and begun to construct the first hedging tools for longevity. The claim of this thesis is to give at first a general overview which instruments was developed in the past and second to test the usefulness through a stochastic simulation of possible future mortality rates of 65 year old austrian males. As decision criteria the calculus of Net Present Value, weighted with an conditional survival probability, was chosen, i.e. a so called Expected Net Present Value. The analysis suggests that the hedge with the European Investmentbank longevity bond, from 2003, would have been possible, at least for 65 year old Austrian males. The Expected Net Present Values was all better off through the purchase of the bond. As second possibility, the hedge through a zero coupon swap, called q-forward and provided by JP-Morgan, was chosen. The results was rather disappointing. The analysis suggests that the price would have been too high, concerning the ruling assumptions of this analysis. The Expected Net Present Values was all worse off through the hedge as without.

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